

# Combining Widening and Acceleration in Linear Relation Analysis

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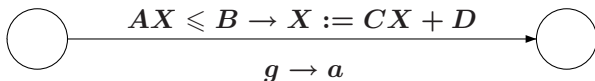


- 1 Introduction and Motivation
- 2 Motivating example
- 3 Simple loops
- 4 Two translation loops
- 5 Implementation - ASPIC

# Introduction

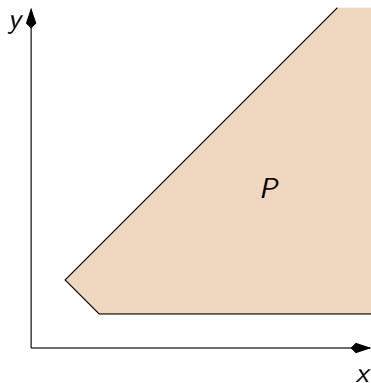
**Context** : Linear Relation Analysis (LRA) [Cousot&Halbwachs 78]  
**The analysis framework**

- CFG with numerical variables and **affine** guards and actions :

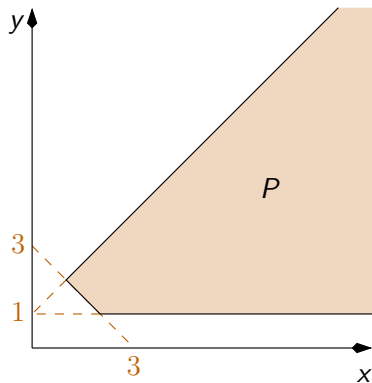


- Safety/non reachability properties
- Forward and/or backward analysis to compute invariants.
- Polyhedral (over-) approximations of invariants sets. (Convex polyhedra in  $\mathcal{P}(\mathbb{Q}^n)$  in **double description**).
- Possibly infinite sequences  $\Rightarrow$  **widening operator**.

# The polyhedral domain (1)

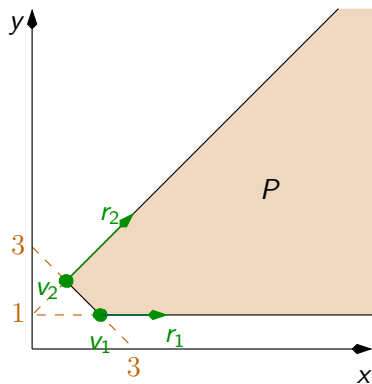


## The polyhedral domain (1)



$$\begin{aligned}
 P &= \{(x, y) \mid \\
 &\quad 1 \leq y \leq x + 1 \wedge x + y \geq 3\} \\
 &= \text{cons}\{AX \leq b\}
 \end{aligned}$$

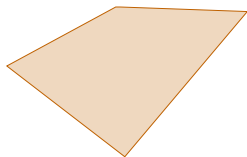
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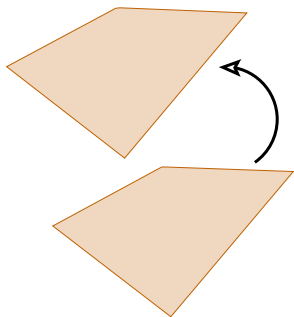
$$\begin{aligned}
 P &= \{\lambda v_1 + (1 - \lambda)v_2 + \mu_1 r_1 + \mu_2 r_2 \mid \\
 &\quad \lambda \in [0, 1], \mu_1, \mu_2 \geq 0\} \\
 &= \text{gen}(V, R)
 \end{aligned}$$

# The polyhedral domain (2)



- **Usual operations** : linear transformation  $P' := CP + D$ , intersection, union (convex hull), test for emptiness.
- Widening :  $P \nabla Q = \text{something}$ .
- Adding rays :  $P \nearrow R = \{x + \sum_{r_j \in R} \mu_j r_j \mid x \in P, \mu_j \in \mathbb{Q}^+\}$

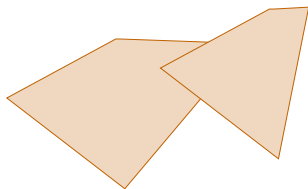
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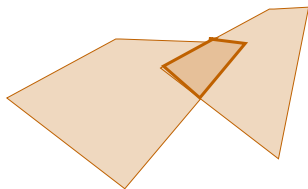


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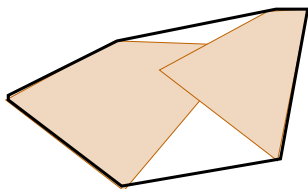
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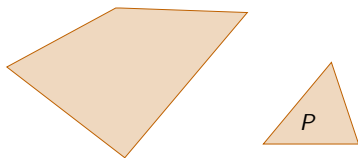
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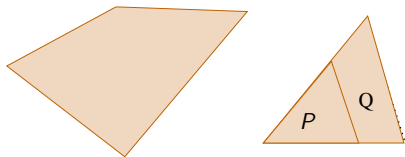
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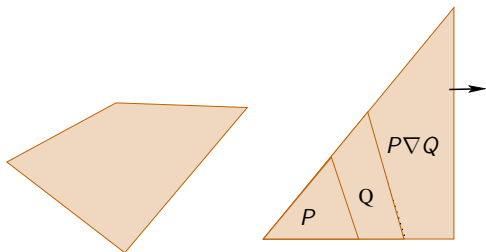
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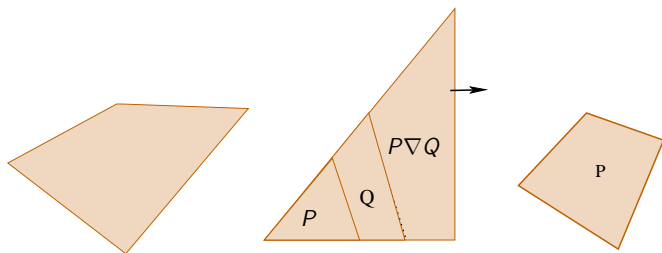
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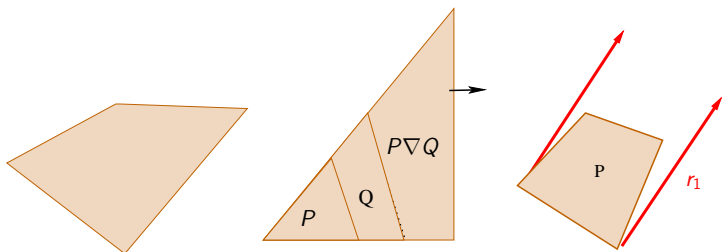
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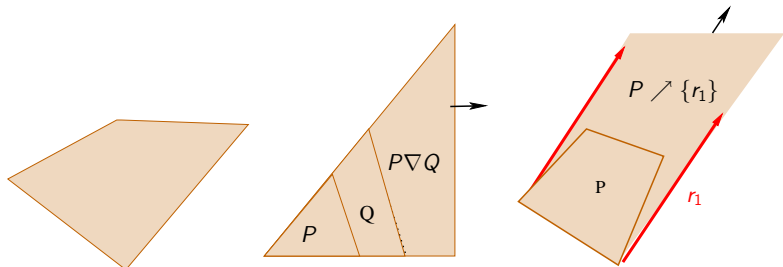
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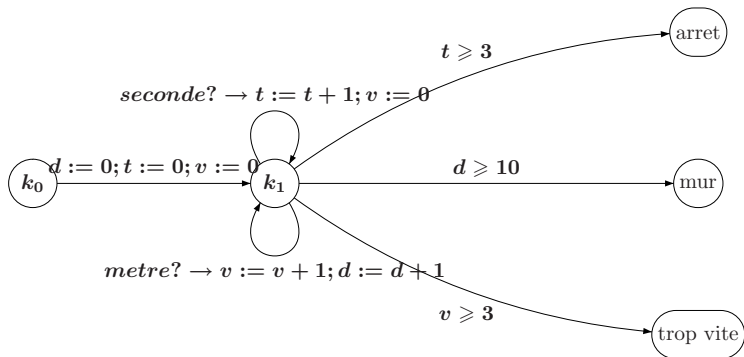


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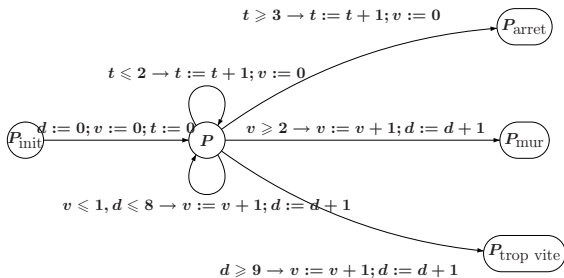
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## LRA : a toy example, the car (1)



- On reçoit les signaux *metre* et *seconde*.
- $d = nb(\textit{metre})$ ,  $t = nb(\textit{seconde})$ ,  $v$  vitesse instantanée.

## LRA : a toy example, the car (1)



(widening upto  $\mathcal{C} = \{v \leq 2, d \leq 9, t \leq 3\}$  et initialisation à  $\perp$ ) :

$$P^1 = \{d = 0; t = 0; v = 0\}$$

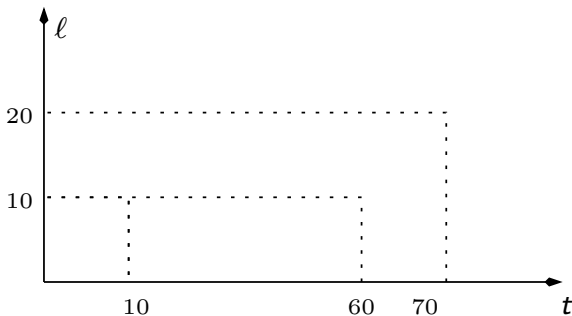
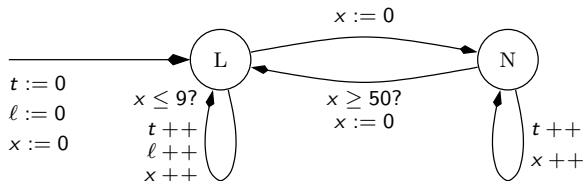
$$P^2 = P^1 \nabla (\{d = t = v = 0\} \sqcup \{d = v = 1; t = 0\} \sqcup \{d = v = 0; t = 1\})$$

$$= \{0 \leq v = d \leq 2; 0 \leq t \leq 3\}$$

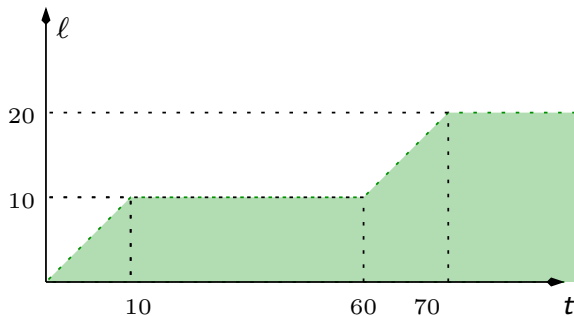
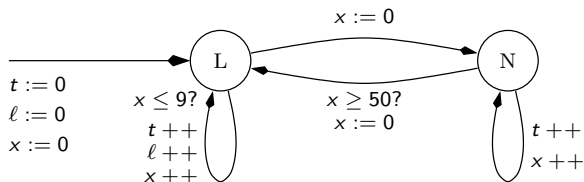
$$P^3 = P^4 = \{0 \leq v \leq d \leq 2t + v; t \leq 3; v \leq 2\}$$

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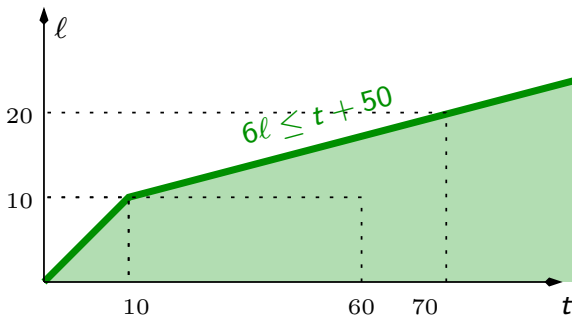
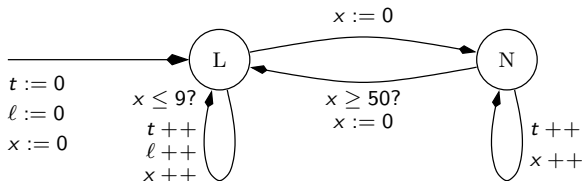
## Example - Gaz Burner - 1



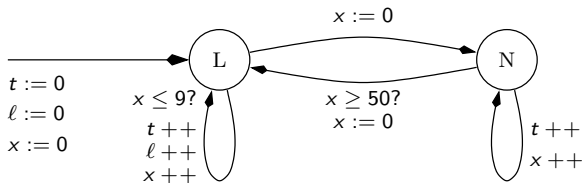
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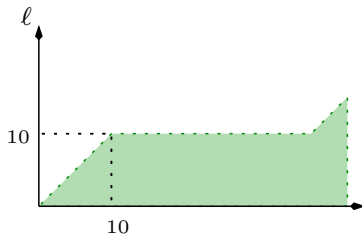
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## Example - Gaz Burner - 2

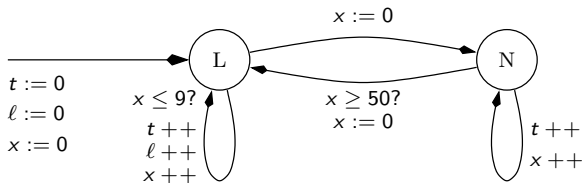


## Standard LRA algorithm

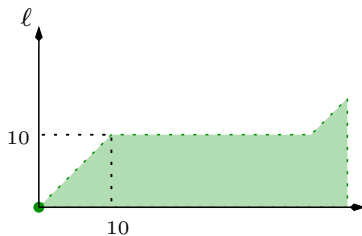




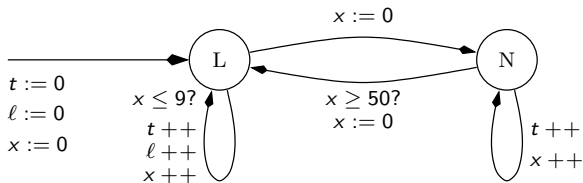
## Example - Gaz Burner - 2



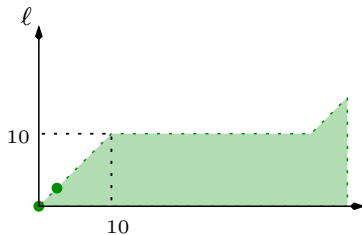
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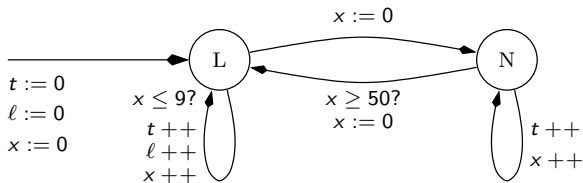
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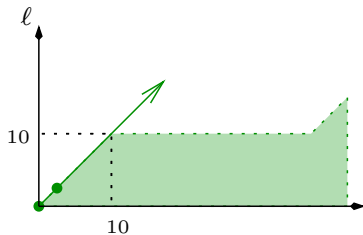
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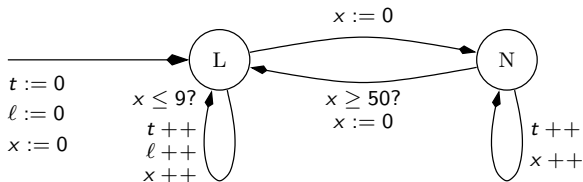
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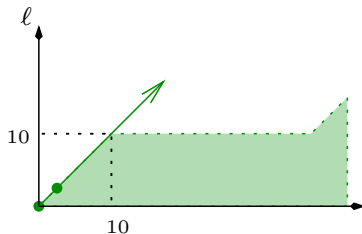
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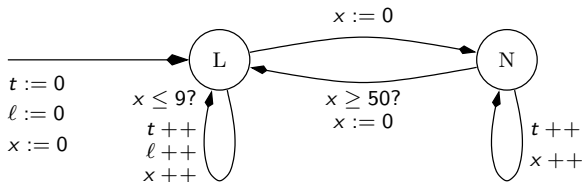


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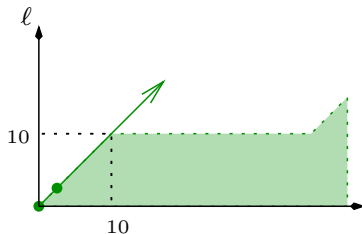


► **lack of precision** : descending sequence will not recover

## Example - Gaz Burner - 2



## Standard LRA algorithm



- ▶ **lack of precision** : descending sequence will not recover
- ▶ **delaying ?** 60 times !

# Existing techniques (1)

## Linear Relation Analysis (LRA) and Widening

- Approximate results
- Possible improvements :
  - **Delaying** the application of the widening, drawback : the cost.
  - **Changing** the operator. [Bagnara&Hill&Zafanella], drawback : no result on the global convergence.
  - Alternating widening and narrowing [Gopan&Reps : CAV 2006], drawback : the cost.

# Existing techniques (2)

## Acceleration

[Boigelot&Wolper, Common&Jurski, Finkel&Sutre&Leroux]

- Computing the **exact effect** of loops on integer sets.
- Encoding : Automata representing Presburger Formula
- **Drawbacks** : restricted class of programs, semi-algorithms, high complexity

▶ Our global objective : “abstract” acceleration **at low cost** for convex polyhedra combined with widening.

▶ More precisely : compute the convex hull of the exact reachable states.

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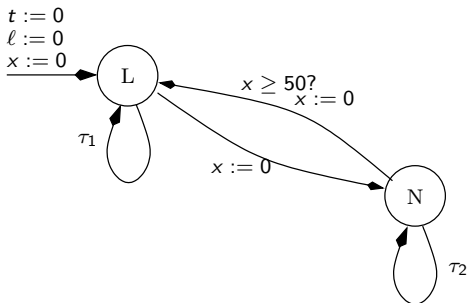
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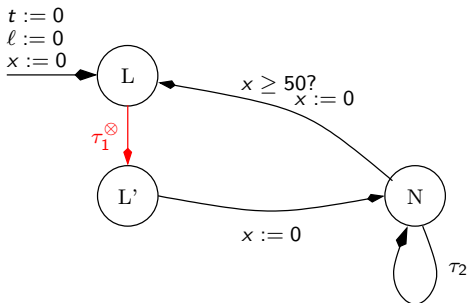
# Acceleration within LRA

We want to replace the loops ( $\tau_i : g_i \rightarrow a_i$ )



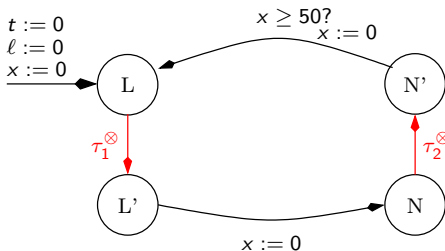
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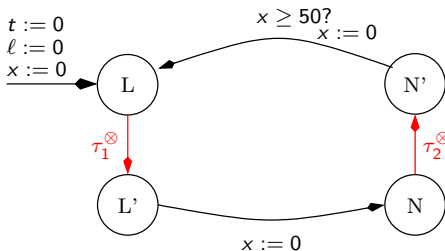
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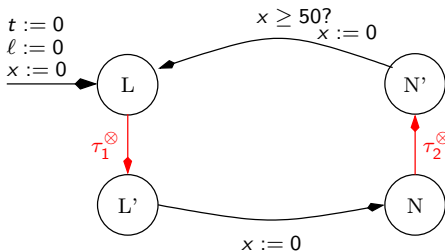
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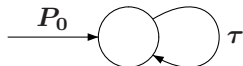
- ▶  $\tau_i^\otimes$  summarizes the effect of **any** number of applications of  $\tau_i$
- ▶ global loop : **accelerate** or **widen**

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## Simple loops

We want to **characterise**  $\tau^*(P_0) = \bigcup_{i \in \mathbb{N}} \tau^i(P_0)$ , with :

$\tau(x) = \text{if } Ax \leq B \text{ then } Cx + D \text{ else } x$



**Previous result** : [Boigelot99, Leroux02] : if  $\exists p, C^{2p} = C^p$  and  $\varphi$  a Presburger set, then  $\tau^*(\varphi)$  is a computable Presburger set.

**New result** (not in the SAS paper) If  $\exists p, C^{2p} = C^p$ , then computing  $\tau^*(P_0)$  reduces itself to computing  $\tau'^*(P'_0)$  :

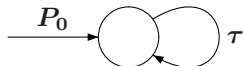
- $\tau'$  a translation/reset transition.
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# Single translation loop

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- (obvious) Proposition

$$\tau^*(P_0) = \{x \mid \exists i \in \mathbb{N}, \exists x_0 \in P_0, \\ Ax_0 \leq B, A(x - D) \leq B, x = x_0 + iD\} \cup P_0$$

## Not convex

- Discrete vs Dense acceleration

$$\tau^{\otimes}(P_0) = \{x \mid \exists i \in \mathbb{Q}^+, \exists x_0 \in P_0, \\ Ax_0 \leq B, A(x - D) \leq B, x = x_0 + iD\} \sqcup P_0$$

## Convex polyhedron

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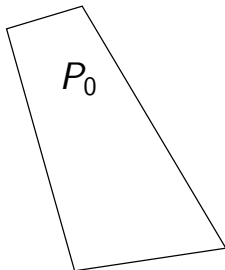
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Computation : Adding rays :

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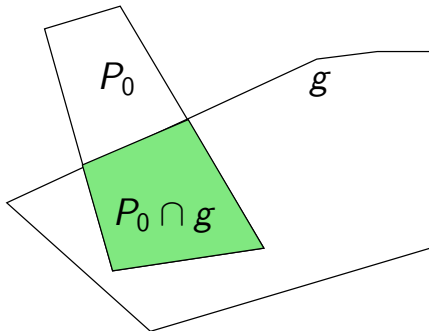
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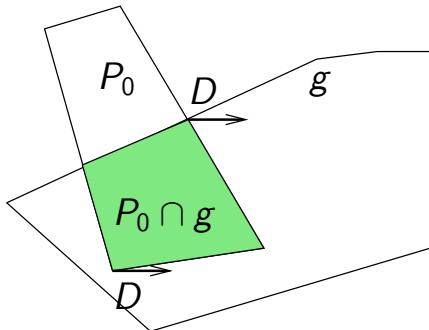
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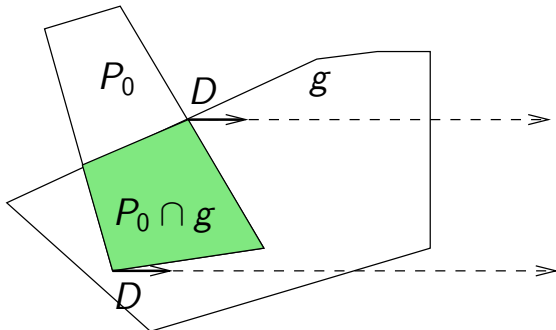
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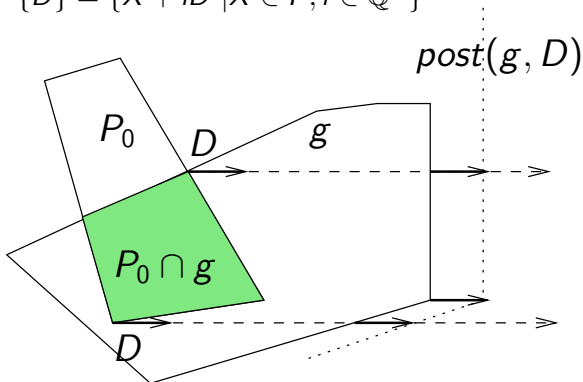
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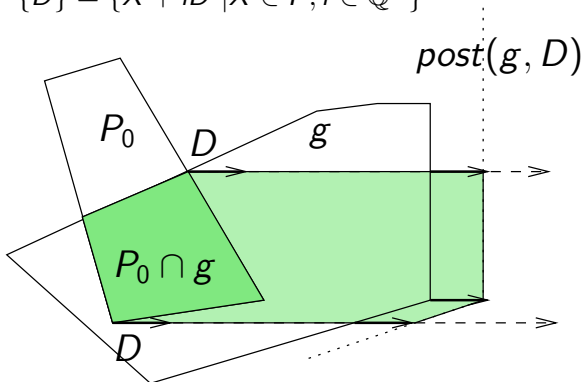
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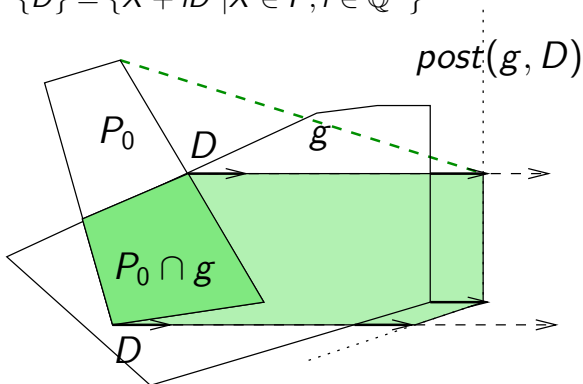
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## Finite Monoids simple loops

What do we lose?

$$\tau = \begin{cases} g = (x \leq 7) \\ a = (x := x + 2) \end{cases}$$

$$P_0 = \{x = 0\}$$

$$\tau^*(P_0) = \{0 \leq x \leq 8\}$$

$$\begin{aligned} \tau^{\otimes}(P_0) &= P_0 \nearrow (1) \cap (x - 2 \leq 7) \\ &= \{0 \leq x \leq 9\} \end{aligned}$$

New result (not in the SAS paper) If  $\exists p, C^p = C^{2p}$ , we can compute an over approximation of  $\tau^*(P_0)$ . We note it  $\tau^{\otimes}(P_0)$ .

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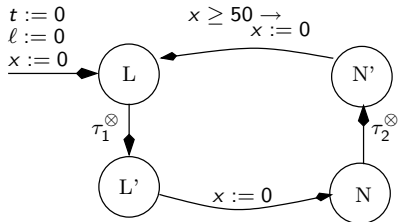
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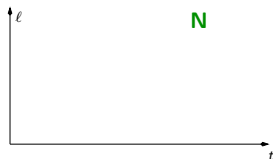
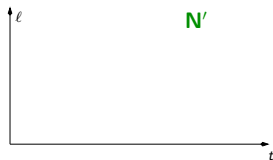
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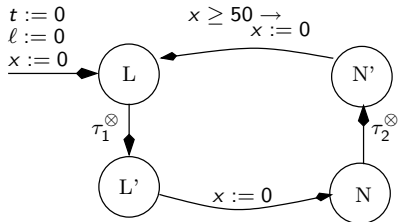
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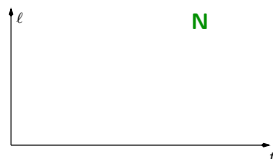
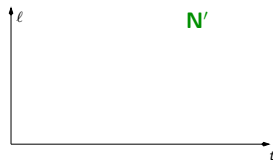
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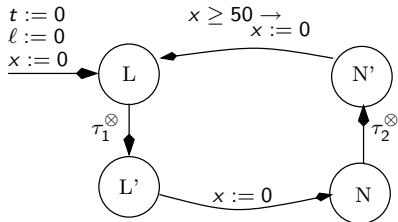
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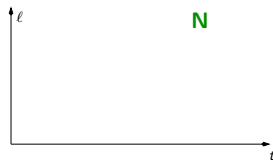
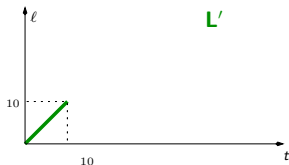
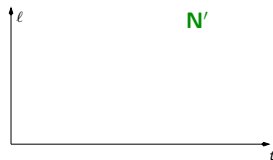
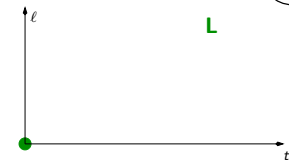
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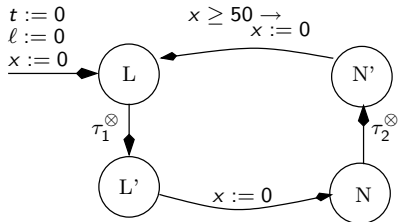


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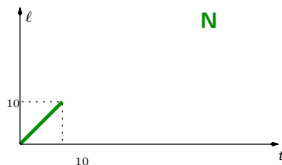
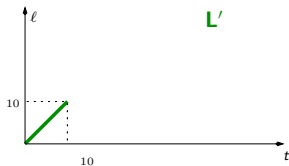
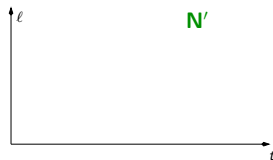




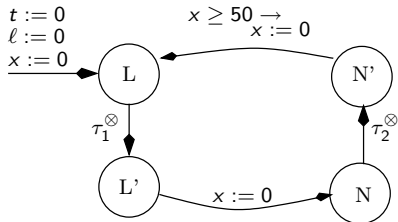
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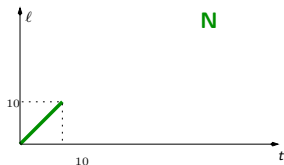
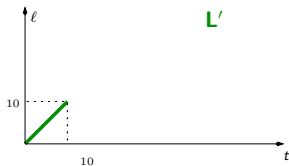
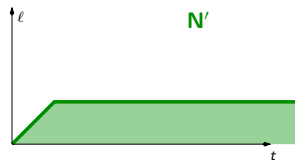
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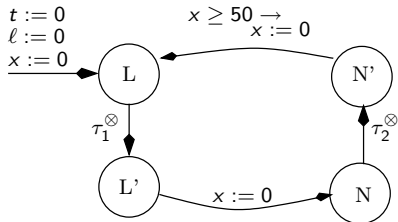
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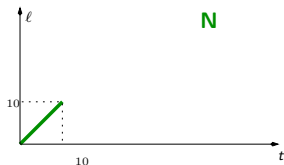
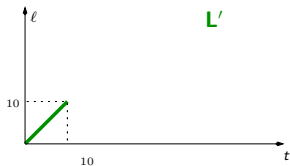
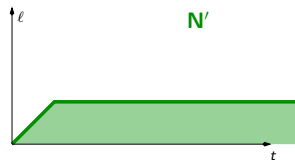
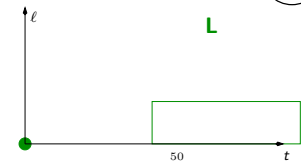
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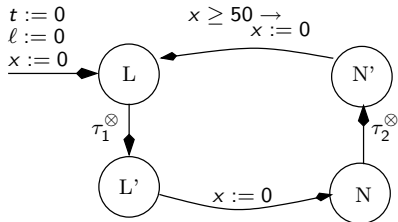
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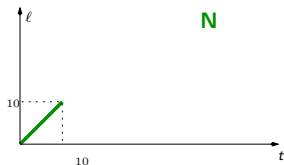
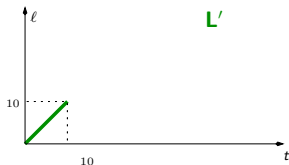
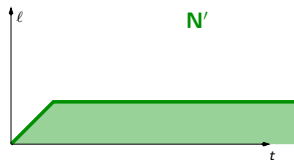
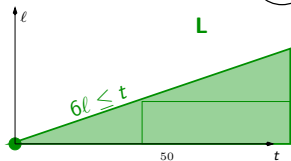
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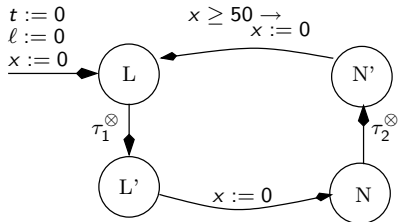
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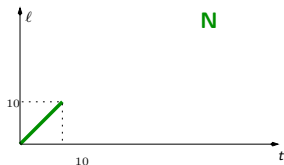
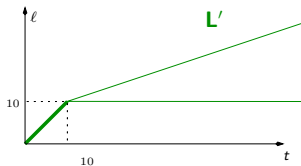
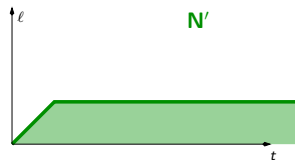
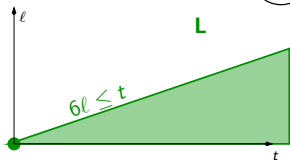
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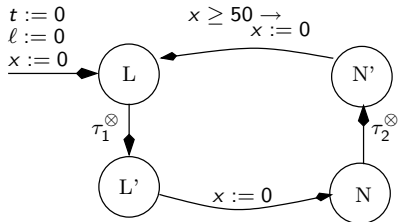
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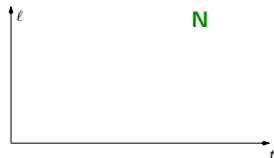
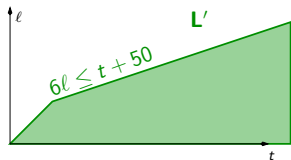
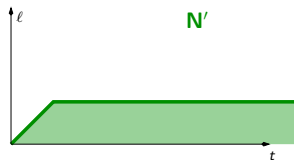
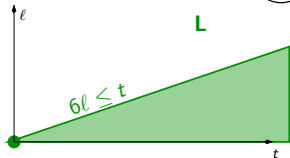
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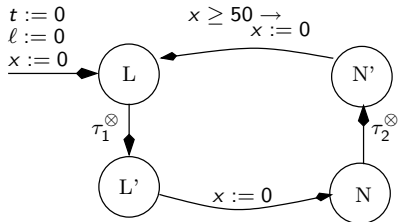
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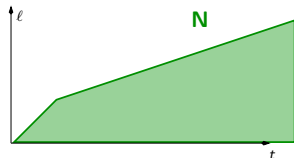
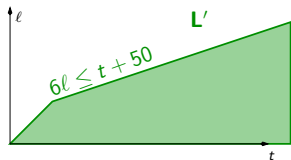
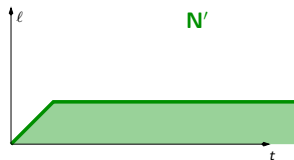
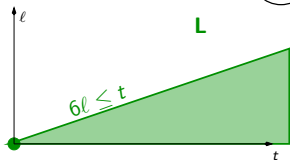
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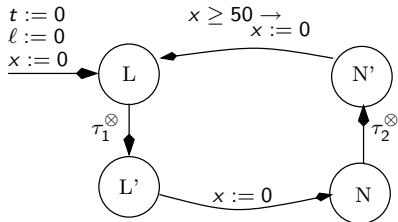
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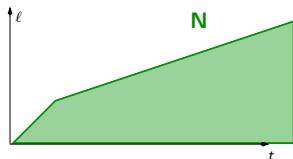
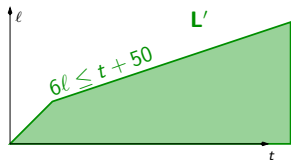
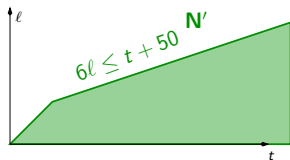
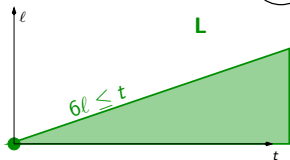
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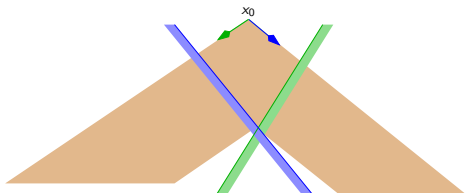




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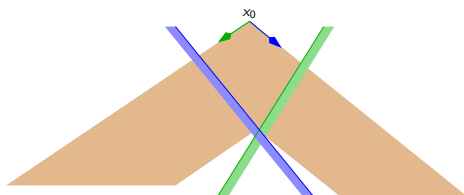
## Two loops - First remarks

$(\tau_1 + \tau_2)^*(P_0)$  is not necessarily **convex** :

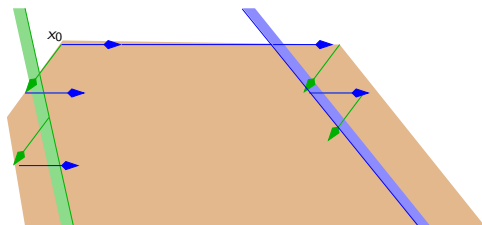


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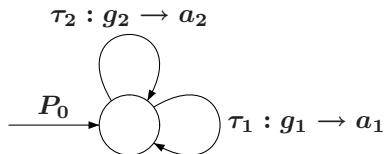
$(\tau_1 + \tau_2)^*(P_0)$  is not necessarily **convex** :



There can be quite complex **oscillations**



## Two simple translation loops – new results



- ▶ **New result** for  $g_i = G_i X_i \leq c_i$  : an algorithm to compute an over-approximation of  $(\tau_1 + \tau_2)^*(P_0)$  (convex polyhedron).
- ▶ In other cases, add a new loop computing  $P_1$  :

$$P_1 = P_0 \sqcup \tau_1^{\otimes}(P') \sqcup \tau_2^{\otimes}(P')$$

where :

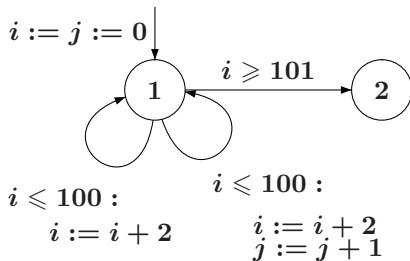
$$P' = (P_0 \cap g_1 \cap g_2 \nearrow \{D_1, D_2\}) \cap g_1 \cap g_2$$

## Old Cousot&amp;Halbwachs78 example

```

i := 0;
j := 0;
while i <= 100 do
1   if ? then i := i + 2
    else i := i + 2; j := j + 1
    fi
od
2

```



Immediate result at control point 1

$$\begin{aligned}
 (0, 0) \nearrow \{(2, 0), (2, 1)\} \cap (i \leq 100) \\
 = 0 \leq 2j \leq i \leq 100
 \end{aligned}$$

# Loops loops with reset (or constant assignments)

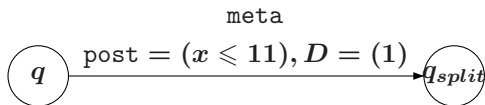
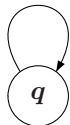
Some kind of loops can be accelerated alltogether.

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# Overview of ASPIC

- A Fixpoint engine for the polyhedral lattice [B. Jeannet]
- Creation of **meta-transitions** if possible :

$$x \leq 10 \rightarrow x++$$



- Classical LRA with forward computation, widening and **meta-transitions**.
- A compilation chain from Lustre, a synchronous functional language designed in Verimag.

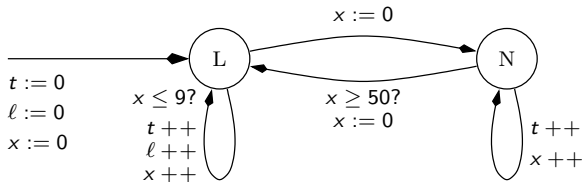


# Aspic demo

ASPIC : Accelerated Symbolic Polyhedral Invariant Computation

<http://www-verimag.imag.fr/~gonnord/aspic/aspic.html>

demo.



# Currently working on

- More general cases of loops.
- Sets of benchmarks coming from :
  - Sensor networks : consumption properties (Maraninchi/Samper)
  - "Programs with list are counter automata" (CAV 06, Iosif/Bozga/Bouajjani/Habermehl/Moro/Vojnar)
  - ...
- My PHD (!)

# Future work

- More general cases of loops.
- Compare/Combine with “Lookahead widening” (CAV 06, Gopan/Reps) Implemented !
- Implement in NBAC tool [B.Jeannet]