Proving Termination of flowcharts programs

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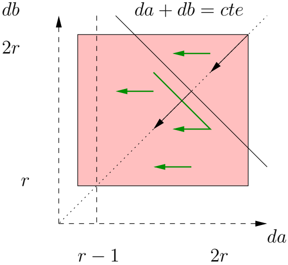
Joint work with Christophe Alias, Alain Darte, and Paul Feautrier (Compsys, ENS Lyon), Lucas Seguinot (ENS Bretagne), David Monniaux (Verimag, Grenoble) and Raphael-Ernani. Rodriguez (Univ Mineas Gerais Brasil).

Context : Transforming WHILE into FOR

Example : GCD of 2 polynomials

```
da = 2r; db = 2r;
while (da >= r) {
  cond = (da >= db || A[expr] == 0);
  if (!cond) {
    tmp = db; db = da; da = tmp - 1;
  } else da = da - 1;
}
```



Hard to optimize for a high-level synthesis tool :

- No loop unrolling possible.
- Limited software pipelining.

▶ Need to bound the number of iterations.

Termination proofs, what for ?

- for fun !
- prove total correctness
- reactive systems need termination of inner loops
- (compute worst-case bounds)

Contributions

Program termination with global multi-dimensional affine rankings :

- Proven to be complete, fully implemented.
- Work on scalability : program-level optimisations, algorithmic optimisation.

► for **general** flowcharts programs (with a proper preprocessing !)

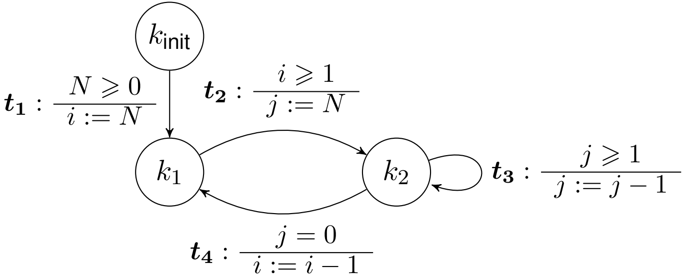
Summary

- From a program : generate an affine automaton with invariants.
- Generate a linear programming (LP) instance and solve it.
- If it is not sufficient, retry in a lesser automaton.

Our model for programs

Interpreted affine automata $(\mathcal{K}, n, k_{init}, \mathcal{T})$

- $\mathcal{K}$: control points
- n rational variables $\boldsymbol{x}$
- $k_{init} \in \mathcal{K}$ the initial control point
- $\mathcal{T}$ the set of transitions (k, g, a, k')



From a Program to an affine Automaton

Sofware Eng.

From program to an accurate model. Semantic transformation.

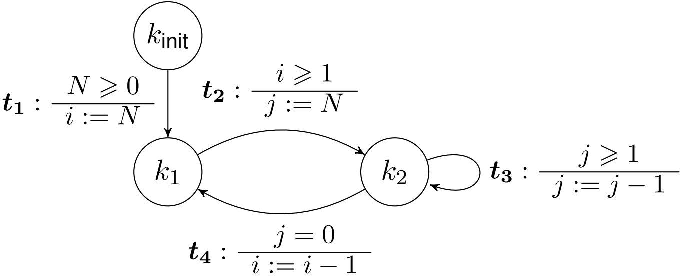
Problem 1 : how to compute such (numerical) automata from general programs ?

- Safe Abstractions for all data structures, arrays, pointers ?
- Handle all C control-flow instructions.
- How to handle programs with functions calls ?

▶ Home-made tool C2FSM or Rose-based tool SToP
TODO : implement inside the LLVM framework.

Invariants

Invariant = formula that over-approximates all the possible values of the variables.



$$k_1 : N \geqslant 0 \wedge i \geqslant 0 \wedge i \leqslant N$$

From an affine Automaton to invariants

Problem 2 : how to compute numerical invariants ?

- Accurate numerical invariants
- Polyhedra
- Within a reasonable amount of time.

We use abstract interpretation.

Tools : ASPIC or the promising Pagai (J. Henry)

Sofware Eng.

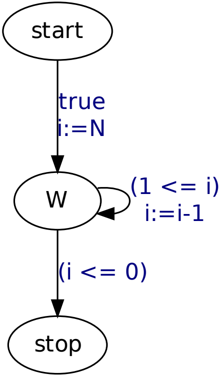
Use compact (internal) representations. Carefully deal with interprocedural analysis.

Ranking functions

A ranking function is :

- A mapping from $(state, value)$ to a well-founded set
- Decreasing (strictly) on each transition.

```
assume(N>0);
i=N;
while(i>0) --i;
```



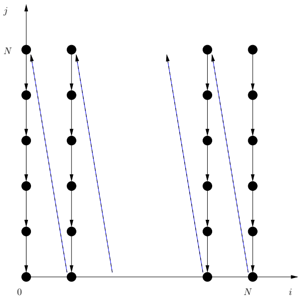
Demo !

Pb and Restrictions

Pb How to compute a ranking function for a given automata ?
We restrict to $\mathbb{N}^p$ with $\leqslant_{lex}$, and **multidimensional** affine rankings :

$$\rho(k, \vec{x}) = A_k.\vec{x} + \vec{b_k}$$

```
//N>0
i = N;
while(i>0)
{
   j = N;
   while(j>0) j--;
   i--;
}
```

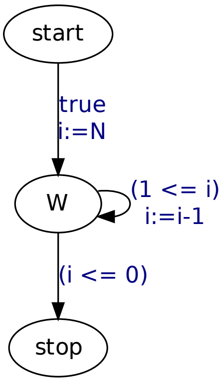


- Demo !

Finding a ranking function - 1

The 1D-case :

```
assume(N>0);
i=N;
while(i>0) --i;
```



Searching for $\alpha_{pc,-} \in \mathbb{Q}$:

$$\begin{array}{rcl}
\rho(start, \vec{x}) & = & \alpha_{start,1}.\mathbf{i} + \alpha_{start,2}.\mathbf{N} \\
 & + & \alpha_{start,3}.\mathbf{i_0} + \alpha_{start,4}.\mathbf{N_0} \\
 & + & \alpha_{start,5} \\
\rho(W, \vec{x}) & = & \alpha_{W,1}.\mathbf{i} + \ldots \\
\rho(stop, \vec{x}) & = & \alpha_{stop,1}.\mathbf{i} + \ldots
\end{array}$$

The constraints are :

- For each control point : $\rho(pc, \vec{x}) \geqslant 0$ on P_{pc}
- For each transition $(\vec{x'} - \vec{x}) \in t \Rightarrow \rho(dest, \vec{x'}) - \rho(src, \vec{x'}) > 0$

Finding a 1D ranking function 2

The 1D-case : incoding into a **linear programming** problem
Constraints for control points : $\rho(pc, \vec{x}) \geqslant 0$ on P_{pc}.

Constraint of the form “linear positive form on a convex polyhedra”▶ Farkas Lemma.

Here (for W) $P_W = \{N_0 > 0, N = N_0, 0 \leqslant i \leqslant N\}$ thus :

$$\begin{array}{rcl} \rho(W, \vec{x}) & = & \lambda_{W,1}.(N_0 - 1) + \lambda_{W,2}.(N_0 - N) \\ & + & \lambda_{W,3}.(N - N_0) + \lambda_{W,4}.i + \lambda_{W,3}.(N - i) \end{array}$$

We were looking for $\rho(W, \vec{x})$ with the following “template” :

$$\rho(W, \vec{x}) = \alpha_{W,1}.i + \alpha_{W,2}.N + \alpha_{W,3}.i_0 + \alpha_{W,4}.N_0 + \alpha_{W,3}$$

▶ Identifying coefficients for **i** : $\alpha_{W,1} = \lambda_{W,4} - \lambda_{W,3}, \ldots$

Finding a 1D ranking function - 3

For decreasing transitions :

$$(\vec{x'} - \vec{x}) \in t \Rightarrow \rho(dest, \vec{x'}) - \rho(src, \vec{x'}) > 0$$

becomes

$$(\vec{x'} - \vec{x}) \in t \Rightarrow \rho(dest, \vec{x'}) - \rho(src, \vec{x'}) \geqslant \epsilon_t$$

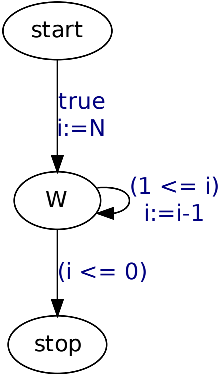
- “some” more constraints.

Objective function : Maximize $\sum_t \epsilon_t$

Finding a 1D ranking function - 4

The 1D-case :

```
assume(N>0);
i=N;
while(i>0) --i;
```



We find :

```
state start:
2+N__o

state W:
1+i

state stop:
0
```

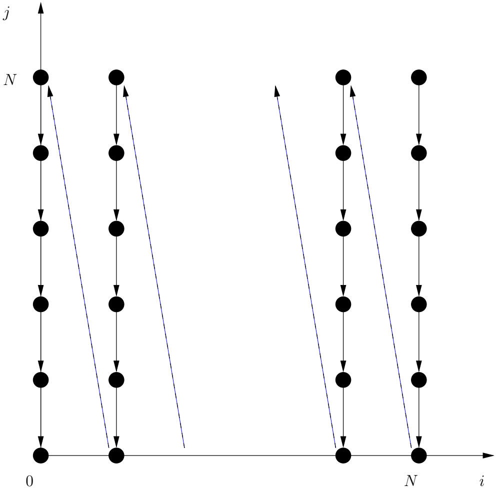
Finding a ranking function - nD

The nD-case, a **greedy algorithm**

- $i = 0$; $T = \mathcal{T}$, set of all transitions.
- While T is not empty do
  - Find a 1D affine function σ, not increasing for any transition, and decreasing for as many transitions as possible.
  - Let $\rho_i = \sigma$; $i = i + 1$; (i^{th} dimension)
  - If no transition is decreasing, **return false**.
  - Remove from T all decreasing transitions.
- $d = i$, **return true**.

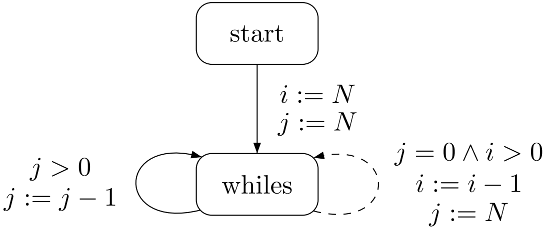
Example - 1

```
//N>0
i = N;
while(i>0)
{
   j = N;
   while(j>0) j--;
   i--;
}
```



Example - 2

```
//N>0
i = N;
while(i>0){
 j = N;
 while(j>0) j--;
 i--;
}
```

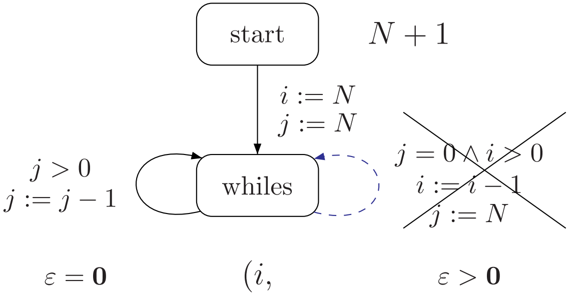


Invariant for `whiles` :

$$\mathbf{-1 < i \leqslant N, -1 < j \leqslant N}, N > 0, N = N_o$$

Example - 2

```
//N>0
i = N;
while(i>0){
 j = N;
 while(j>0) j--;
 i--;
}
```



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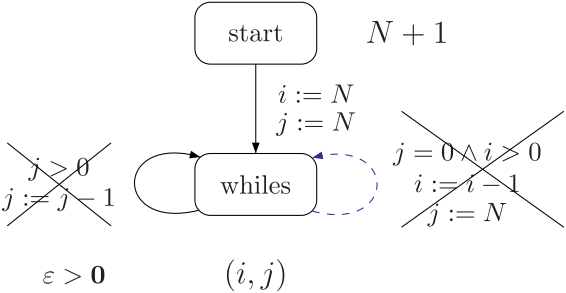

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Example - 2

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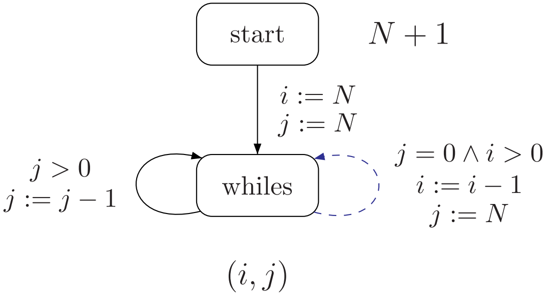

Invariant for `whiles` :

$$\mathbf{-1 < i \leqslant N, -1 < j \leqslant N}, N > 0, N = N_o$$

In summary

From (arbitrary) flowchart programs :

- Compute an affine abstraction.
- Compute invariants on each control point.
- Compute and solve linear programming problems from the graph and its invariants.

This algorithm (+ completeness results) and preliminary experimental results have been published in Static Analysis Symposium (Alias et al, 2010).

Our toolsuite

1. C2FSM for the front-end
2. ASPIC for the invariants
3. RANK for the computation of the ranking function.

Published in [TAPAS2010] and [CSTV 2012].

Available for demo at the url :

`http://compsys-tools.ens-lyon.fr/`

Some experimantal results

Sorting arrays :

Name	LOCs	Time(c2fsm/analysis) [1]	dim
selection	20	1.0/0.4	3
insertion	12	0.6/0.22	3
bubble	22	1.2/0.4	3
shell	23	1.0/1.1	4
heap	45	3.0/2.8	3

1. user time in seconds on a Pentium 2GHz with 1Gbyte RAM

Some comments on experimental results

- The algorithm works well on small challenging programs from the litterature.
- The form of the automaton has a strong impact on the invariants.
- The precision of invariants is **crucial**.

But the size of the LP instances grows exponentially and the solvers cannot deal with too much variables

```
ex2 : 10 loc / automaton : 10 vars, 5 transitions
-- > 3LP, average  180L/75 cols
heapsort : 30 loc / automaton : 12 vars, 10 transitions
--> fail.
```

2 ways of improvement

Two main directions of work :

- Divide and conquer : slice, cut, and go.
- Work on the 'pratical' complexity of the initial algorithm.

Divide and conquer 1/2

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Work on smaller instances of programs.

We use classical (static methods for safety) :

- slicing : we designed a specialized slicing for termination
- compute **context information**
- cut into **kernels** with preconditions
- prove termination on kernels.

► With C. Alias and G. Andrieu. Research report and tool paper SToP [TAPAS 2012]

Divide and conquer 2/2

```
if(up == 0){
  i = 1;
  while(i <= n){
     a[i] = a[i+n];
     i = i+1;
  }
}
```

(a) Before slicing

```
if (up == 0) {
  i = 1;
  while(i <= n){
     i = (i + 1);
  }
}
```

(b) After slicing

Work on the initial algorithm 1/2

Even after slicing all programs are not tractable with the first (monodimensional) algorithm.

Lesson - Limits on Soft. Eng. :-)

Sometimes, working on the initial algorithm itself is the only solution.

- Idea 1 : work only on cutsets and on a compact version of the graph (Henry/Monniaux)
- Idea 2 : Construct incrementaly the (dual) LP programs with counter examples computed with an SMT-solver. The size of LP programs does not depend on the complexity of the transitions.

▶ With D. Monniaux and L. Seguinot. Submitted to ESOP'14.

Work on the initial algorithm 2/2

First experimental results :

Example	Ranking function	Rank tool		Smterm			
		#LP	Avg. #lines/#cols	#LP	Avg. #lines/#cols	#SMT	Avg. size
$easy1$	$41 - x$	1	$334/155$	1	$3/6$	3	21
$easy2$	z	2	$86/42$	1	$3/5$	3	16
$wcet2$	$-11i - j + 65$	2	$225/94$	2	$4/6$	4	20
$exmini$	$102 - i - j + k$	2	$140/65$	3	$6/8$	5	16
$cousot9$	$\begin{pmatrix} i \\ j \end{pmatrix}$	3	$180/75$	5	$6/8$	6	25

Conclusion 1/2

Biblio

A survey on termination techniques : [Ben Amram 2012].

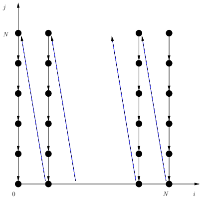
Current / Future work :

- Implement and validate the SMT-based approach.
- Extract kernels from big programs and contribute to termination benchmarks.
- Non termination preconditions.
- Cooperation between techniques : computing invariants and proving termination at the same time ?

Conclusion 2/2

Back to the initial problem :

- Generate for loops (source-to-source transformation).
- Link with program scheduling ?



Key idea

Finding a ranking function is nearly the same as finding a (sequential) schedule.