Program verification

introduction, bounded-model checking, SAT, symbolic model checking

Laure Gonnord David Monniaux

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The teaching staff

- Laure GONNORD, associate professor, LIP laboratory, University of Lyon
  `Laure.Gonnord@ens-lyon.fr`
- David MONNIAUX, CNRS senior researcher, VERIMAG laboratory, Grenoble
  `David.Monniaux@imag.fr`

Safe software ?

- safety-critical software (control of vehicles e.g. airplanes and cars, surgical robots, radiation therapy...)
- less critical software (flight management systems, financial transactions, unmanned spacecraft...)
- general-purpose software?
- (hot topic) software facing the Internet, newer security and privacy issues (e.g. 0-day vulnerabilities sold to intelligence services; are lives at stake ?)

Software engineering

Considers the means of production of software:

- documentation imperatives
- organization of software development teams
- good programming practices
- use of appropriate programming languages
- software development environments

Try to reduce the number of errors at the source.

“Software metrics”
Use of `lint`-like tools or more advanced

Not covered in this course

Proving properties of software

- Basic idea: software has **mathematically defined behaviour**
- Possible to do mathematical **proofs** on software
- Possible to **automate these proofs**

Program proofs

Proving that software truly does what it is meant to do.

behaviours $\subseteq$ acceptable behaviours

- What does software do?
- What is it meant to do?
- What is a proof?

Semantics

A **precise definition** of what a program does — given for all programs within a programming language.

Very difficult for a full industrial language e.g. C++

vs

A definition in $\pm$ vague **natural language** (e.g. ISO C and C++ standards, programming language manuals...)

Imprecise, fuzzy, sometimes contradictory definitions. Language lawyers. Endless discussions on what a program should or should not be doing, on what a compiler has the right to do or not.

Specification

What software should do

- informal definition in natural language
- formal mathematical definition

Is the specification consistent?

Difficulties in writing specifications:

- Are all requirements taken into account?
- Redundancy with implementation

Specification example: `sort`

Unix command `sort`

(Without the options) Simple informal specification: “sort the lines in a file”

In more detail: complicated — e.g

- what is the sorting order wrt non-ASCII characters?
- how are equivalent lines sorted (e.g. numeric ordering)

Mathematical definition possible, but long.

Difficulty

behaviours $\subseteq$ acceptable behaviours

Both sets are not well defined in general.

May need to fix

- language definition
- target compilation environment (evaluation order, size of basic types, alignment...)
- precise specification

How about proofs?

The Halting Problem

Simple language: integers ($\mathbb{Z}$), tests, loops

There is no algorithm that says, given a program, whether this program halts. (Turing)

The Halting Problem, proof

Suppose we have a "magical analyzer" A: answer $A(P, X) = 1$ "program P terminates eventually on input X" $A(P, X) = 0$ otherwise

```
int B(Program x) {
  if (A(x,x)==0) {
    return 1;
  } else {
    while(true) {}
  }
}
```

What is $B(B)$? (B applied to its own source code)

The Halting Problem, contradiction

```
int B(Program x) {
  if (A(x,x)==0) {
    return 1;
  } else {
    while(true) {}
  }
}
```

If $B(B) = 1$ then $A(B, B) = 0$ "program B does not terminate on input B". Absurd!

If $B(B)$ loops then $A(B, B) = 1$ "program B terminates on input B". Absurd!

There is no magical static analyser.

Workarounds

What is impossible is to check reachability

1. automatically
2. without false positives
3. without false negatives
4. on systems of unbounded state
5. with unbounded execution time

Lifting restrictions opens possibilities!

Starting states + transitions

State of the program / of the machine = values of variables, registers, memories…within Σ.

Par exemple :

- if system state = 17 Booleans, then $\Sigma = \{0,1\}^{17}$;
- if system state = 3 unbounded integers, then $\Sigma = \mathbb{Z}^3$;
- if finite automaton, Σ is the set of states;
- if stack automaton, state = pair (automaton state, stack contents), so $\Sigma = \Sigma_S \times \Sigma_P^*$.

Transition relation $\rightarrow$: $x \rightarrow y$ = “if I’m at x I can go to y at the next step”

Safety properties

(A more general definition exists. Consider the simplest case.)

Show that a program cannot reach a "bad state" (crash, out-of-specification).
Set W of bad states.

Show that there is no $n \geq 0$ and $\sigma_0 \to \sigma_1 \to \dots \sigma_n$, σ_0 initial state (= reset), $\sigma_n \in W$ (trace of n steps leading to a bad state).

Otherwise said: $\sigma_0 \to^* \sigma_n \in W$. $\to^*$ **reflexive transitive closure** of $\to$.

Reachable states

Let $\Sigma_0 \subseteq \Sigma$ be the initial states.
The set A of **reachable states** is the set of states σ such that TODO

$$\exists \sigma_0 \in \Sigma_0 \; \sigma_0 \rightarrow^* \sigma \tag{1}$$

On veut montrer que $A \cap W = \emptyset$.

Bounding the state space

Restrict to a finite number of variables of a finite type.

Finite state space $\implies$ "it's just a big finite automaton!"

Everything is decidable!

Explicit-state model checking

Given a transition relation τ

- Set $R := \{\text{initialstate}\}$
- For each state x in R, add all x' such that $(x, x') \models \tau$
- Do it until R is saturated (no new states are added)
- Then R is the set of **reachable states**.

Then test whether R contains undesirable states.

Implementation issues

If state = n Boolean variables, 2^n possible states.

Memory usage linear in number of reachable states.

Store states in hash table.
Store states in distributed hash table.

Tool example: CADP (INRIA Grenoble)

Explicit state model checking, a weakness

Representation expensive even if the set of reachable states is "simple".
e.g. $\{0,1\}^n$ "everything reachable" needs $\Theta(2^n)$ memory

Try to compress sets of states by **symbolic** representation.

Reachable states as a limit

X_n is the set of states reachable within n steps of $\rightarrow$: $X_0 = \Sigma_0$, $X_1 = \Sigma_0 \cup R(\Sigma_0)$, $X_2 = \Sigma_0 \cup R(\Sigma_0) \cup R(R(\Sigma_0))$, etc.

with $R(X) = \{y \in \Sigma \mid \exists x \in X\; x \rightarrow y\}$.

X_k grows wrt $\subseteq$.
Its limit (= union of all terms) is the **set of reachable states**.

Iterative computation

Remark $X_{n+1} = \Sigma_0 \cup R(X_n)$.

Intuition: to reach in at most $n+1$ steps

- either in 0 steps = initial states Σ_0
- either in $0 < k \leq n+1$ steps, thus in at most n steps (X_n) followed by another step

But how to efficiently represent the X_n and compute over them?

The problem

Representing **compactly** sets of Boolean states

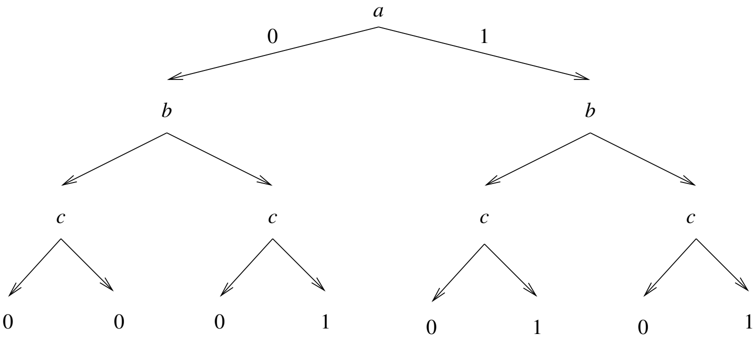
A set of vector n Booleans = a function from $\{0,1\}^n$ into $\{0,1\}$.

Example: $\{(0,0,0),(1,1,0)\}$ represented by $(0,0,0) \mapsto 1$, $(1,1,0) \mapsto 1$ and 0 elsewhere.

Expanded BDD

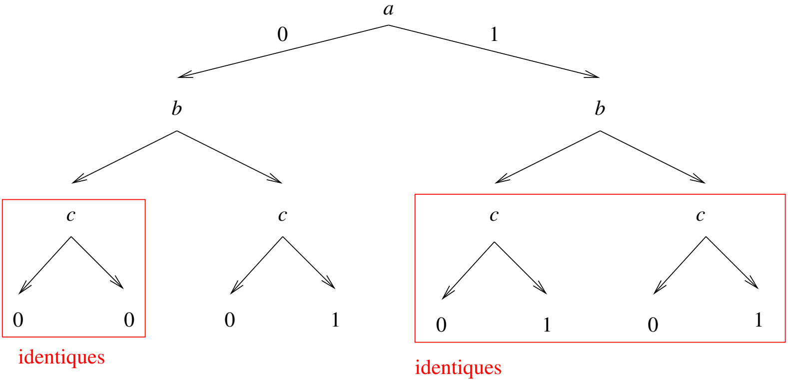
Binary decision diagrams

Given ordered Boolean variables (a, b, c), represent $(a \wedge c) \vee (b \wedge c)$:

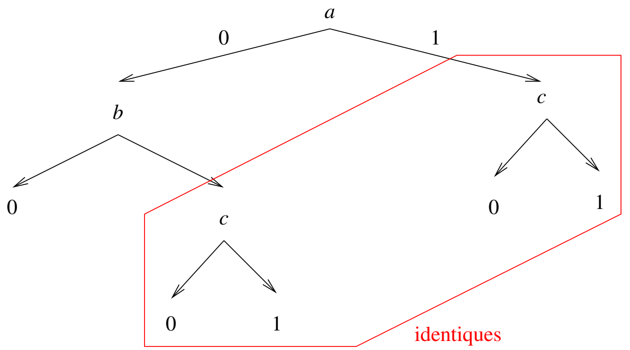


Removing useless nodes

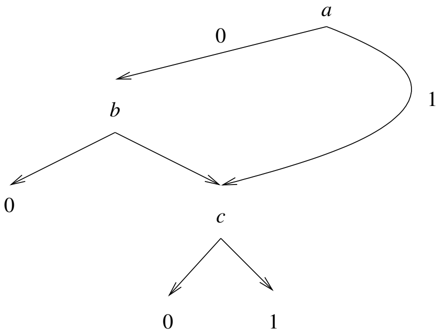
Silly to keep two identical subtrees:



Compression



Reduced BDD



Idea: turn the original tree into a DAG with **maximal sharing**.

Two different but isomorphic subtrees are never created.
Canonicity: a given example is always encoded by the same DAG.

Implementation: hash-consing

Important: implementation technique that you may use in other contexts

“Consing” from “constructor” (cf Lisp : cons).

Keep a **hash table** of all nodes created, with hashcode $H(x)$ computed quickly.

If node = (v, b_0, b_1) compute H from v and unique identifiers of b_0 and b_1

Unique identifier = address (if unmovable) or serial number

If an object matching (v, b_0, b_1) already exists in the table, return it

How to collect garbage nodes? (unreachable)

Garbage collection in hash consing

Needs **weak pointers**: the pointer from the hash table should be ignored by the GC when it computes reachable objects

- Java `WeakHashMap`
- OCaml `Weak`

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(Other use of weak pointers: caching recent computations.)

Hash-consing is magical

Ensures:

- **maximal sharing:** never two identical objects in two $\neq$ locations in memory
- ultra-fast equality test: sufficient to **compare pointers** (or unique identifiers)

And once we have it, BDDs are easy.

BDD operations

Once a variable ordering is chosen:

- Create BDD false, true(1-node constants).
- Create BDD for v, for v any variable.
- Operations $\wedge$, $\vee$, etc.

Binary BDD operations

Operations $\wedge$, $\vee$: recursive descent on both subtrees, with **dynamic programming**:

- store values of $f(a, b)$ already computed in a hash table
- index the table by the unique identifiers of a and b

Complexity with and without dynamic programming?

Binary BDD operations

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Complexity with and without dynamic programming?

- without dynamic programming: unfolds DAG into tree $\Rightarrow$ exponential
- with dynamic programming $O(|a|.|b|)$ where $|x|$ the size of DAG x

Quantifiers

BDD for formula F over variables x, y, z.
Want a BDD for formula $\exists x\, F$ over variables y et z.
$[\exists x\, F](y, z) \equiv F(0, y, z) \vee F(1, y, z)$: compute $F[0/x] \vee F[1/x]$
($F[b/x]$ is F where x has been replaced by b).

Same for $\forall$ but with $\wedge$.

Otherwise said **quantifier elimination**.

Back to transition systems

- The set Σ_0 of initial states is defined by a formula over $x_1, \ldots, x_n \Rightarrow$ a BDD over n variables.
- The transition relation T over Boolean variables $x_1, \ldots, x_n, x'_1, \ldots, x'_n$ (x' = updated x) $\Rightarrow$ a BDD over $2n$ variables.

Recall $\phi(X) = \Sigma_0 \cup R(X)$, in formulas:

$$\phi(X) = \Sigma_0 \vee (\exists x_1, \ldots, x_n (X \wedge T))[x'_1/x_1, \ldots, x'_n/x_n] \qquad (2)$$

All operations doable on BDDs!

Iterative computations over BDDs

Compute sequence $X_0, \ldots$ with $X_0 = \Sigma_0$ and $X_{n+1} = \phi(X_n)$, stop when $X_n = X_{n+1}$ (recall: ultra-fast equality test!)

(Or stop when X_i intersects bad states.)

Sounds very simple
but many possible optimizations and variants (e.g. **signed BDDs**), much work needed

In practice, need other operators (e.g. "constrain", "restrict"...)

Backward analysis

TODO

- **Forward:** compute reachable states by $\rightarrow$ from initial states Σ_0, test intersection with bad states W
- **Backward:** compute co-reachable states from W, test intersection with initial states Σ_0

A glimpse into the next weeks

The set X of reachable states satisfies $X = \phi(X)$.
It is the least set wrt $\subseteq$ that satisfies $\phi(X) \subseteq X$
$\phi(X)$ = "next states from X and add initial states"

Search for X satisfying $\phi(X) \subseteq X$ (**inductive invariant**).

If **any** inductive invariant does not intersect W, W unreachable.

Industrial use: hardware

Clocked hardware $\simeq$ reset state + transition relation

Checking properties of circuits during conception (building prototypes is very expensive)

Tools such as Cadence-SMV

Bounded model checking

BDDs are too costly (worst-case exponential time and space)

Unbounded reachability in Boolean circuits is PSPACE-complete

Idea: limit search to *n* steps, "only" NP-complete