Program verification

Data-flow analyses, numerical domains

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Context

Program Verification / Code generation:

- Variables : value range, scope, lifespan, constants, . . .
- Arrays : illicit accesses, alias discovery. . .
- Data Structures : memory leaks, null pointer dereferences. . .

▶ static analyses, of different kinds

Plan

Data Flow analysis

- Available expressions - Recall from Compiler Course
- Live Variable analysis
- Toward a generalisation of these analyses

Abstract Interpretation

- Transition systems and invariants
- Computing Invariants (forward)
- Non-relational vs relational analyses

Linear Relation Analysis

- Classical Linear Relation Analysis
- Some improvements
- Diverse use of AI

Tools

Additional Material

What for ?

Avoiding the computation of an (arithmetic) expression :

```
x:=a+b;
y:=a*b;
while(y>a+b) do
   a:=a+a;
   x:=a+b;
done
```

Some defs

Definition

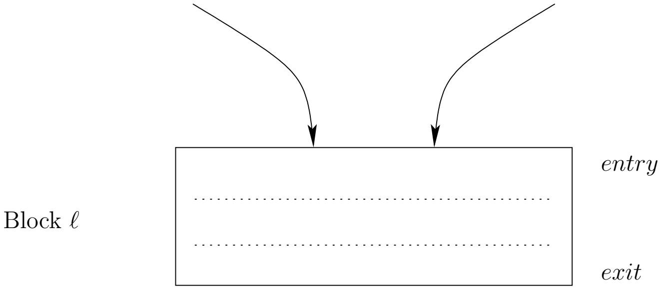
An expression is **killed** in a block if any of its variables is used in the block.

Definition

A **generated** expression is an expression evaluated in the block and none of its variables is killed in the block.

- Sets : $kill_{AE}(block)$ and $gen_{AE}(block)$

Data flow expressions

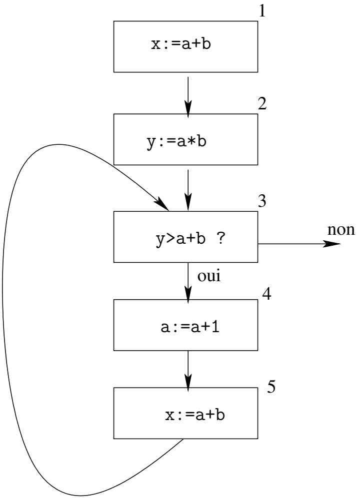


$$AE_{entry}(\ell) = \begin{cases} \emptyset & \text{if } \ell = init \\ \bigcap\{AE_{exit}(\ell') \mid (\ell', \ell) \in flow(G)\} & \end{cases}$$

$$AE_{exit}(\ell) = \left(AE_{entry}(\ell) \setminus kill_{AE}(\ell)\right) \cup gen_{AE}(\ell)$$

On the example - equations

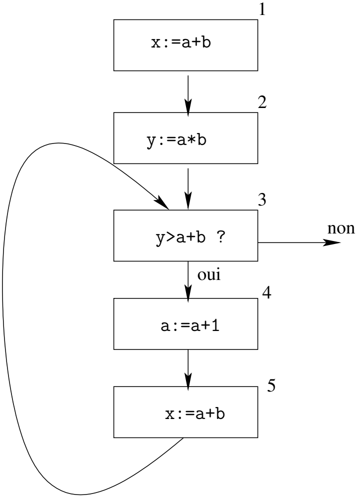
ℓ	$kill_{AE}(\ell)$	$gen_{AE}(\ell)$
1	$\emptyset$	{a+b}
2	$\emptyset$	{a*b}
3	$\emptyset$	{a+b}
4	{a+b, a*b, a+1}	$\emptyset$
5	$\emptyset$	{a+b}



On the example - final solution

ℓ	$AE_{entry}(\ell)$	$AE_{exit}(\ell)$
1	$\emptyset$	{a+b}
2	{a+b}	{a*b, a*b}
3	{a+b}	{a+b}
4	{a+b}	$\emptyset$
5	$\emptyset$	{a+b}

- a+b is available on entry to the loop, not a*b
- Improvement of code generation



Another example : live ranges

```
x:=2;
y:=4;
x:=1;
if (y>x) then z:=y else z=y*y ;
x:=z;
```

Definition

A variable is **live** at the exit of a block if there exists a path from the block to a use of the variable that does not redefine the variable.

Problem : determine the set of variables that *may be* live after each control point.

Data flow expressions

Definition

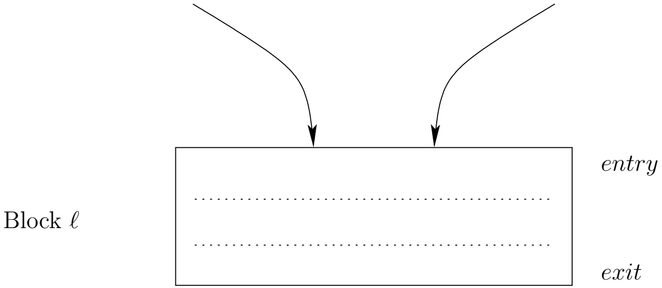
A variable that appears on the left hand side of an assignment is **killed** by the block. Tests do no kill variables.

Definition

A **generated** variable is a variable that appears in the block.

- Sets : $kill_{LV}(block)$ and $gen_{LV}(block)$

Data flow expressions



$$LV_{exit}(\ell) = \begin{cases} \emptyset & \text{if } \ell = \mathit{final} \\ \bigcup\{LV_{entry}(\ell') | (\ell', \ell) \in \mathit{flow}(G)\} \end{cases}$$

$$LV_{entry}(\ell) = \left(LV_{exit}(\ell) \backslash \mathit{kill}_{LV}(\ell)\right) \cup \mathit{gen}_{LV}(\ell)$$

Final result and use

Backward analysis and we want the smallest sets, here is the final result : (we assume all vars are dead at the end).

ℓ	$LV_{entry}(\ell)$	$LV_{exit}(\ell)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
5	$\{y\}$	$\{z\}$
5	$\{y\}$	$\{z\}$
5	$\{z\}$	$\emptyset$

- Use : Dead code elimination ! Note : can be improved by computing the use-defs paths. (see Nielson/Nielson/Hankin)

Common points

- Computing growing sets from $\emptyset$ via *fixpoint iterations*. (or the dual)
- Sets of equations of the form (collecting semantics) :

$$SS(\ell) = \bigcup_{(\ell',\ell)\in E} f(SS(\ell'))$$

  where f is computed w.r.t. the *program statements*

- SS is an **abstract interpretation** of the program.

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- Additional Material

Goal

Propagating **information** about program variables (numerical,arrays, . . .) in order to get **invariants**.

- We focus on **numerical variables** here.

Initial states + transitions

Program or machine state = values of variables, registers, memories… within state space Σ.

Examples:

- if system state = 17-bit value, then $\Sigma = \{0,1\}^{17}$;
- = 3 unbounded integers, $\Sigma = \mathbb{Z}^3$;
- if finite automaton, Σ is the set of states ;
- if stack automaton, complete state = pair (finite state, stack contents), thus $\Sigma = \Sigma_S \times \Sigma_P^*$.

Transition relation $\rightarrow$ $x \rightarrow y$ = "if at x then can go to y at next time".

Reachable states

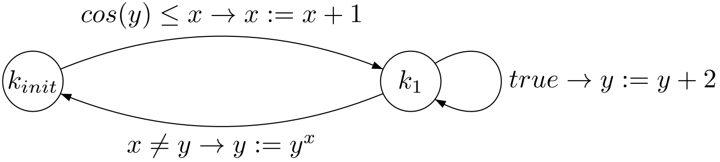
Let $\Sigma_0 \subseteq \Sigma$ the set of initial states of the program.
The **reachable** states are obtained by successively applying the transition relation, hence σ is reachable iff :

$$\exists \sigma_0 \in \Sigma_0 \ \sigma_0 \rightarrow^* \sigma$$

We also define X_n as the set of states reachable in at most n turns : $X_0 = \Sigma_0$, $X_1 = \Sigma_0 \cup R(\Sigma_0)$,
$X_2 = \Sigma_0 \cup R(\Sigma_0) \cup R(R(\Sigma_0))$, etc.

with $R(X) = \{y \in \Sigma \mid \exists x \in X \ x \rightarrow y\}$.
The sequence X_k is ascending for $\subseteq$. Its limit ($=$ the union of all iterates) is the **set of reachable states**.

Reachable states for programs



Semantics of the programs as **transition systems** :

- A **state** is a pair (pc, Val) :

$$\mathsf{Val} : \mathsf{Var} \rightarrow \mathcal{N}^d$$

  - Var is $[\![0, \ldots, d-1]\!]$ (finite set, d vars)
  - $\mathcal{N}$ is $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$
- **Initial** states : $(pc_0, allv)$.

Iterative computation

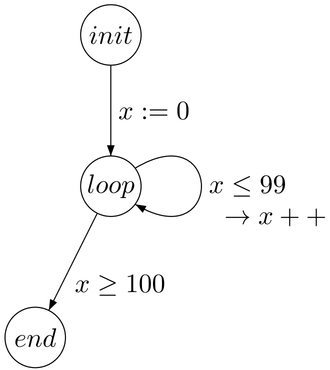
Remark $X_{n+1} = \phi(X_n)$ with $\phi(X) = \Sigma_0 \cup R(X)$.

How to **compute efficiently** the X_n? And the limit?

- Explicit representations of X_n (list all states) : If Σ finite, X_n converges in at most $|\Sigma|$ iterations.
- else, we have to cope with two problems :
  - Representing the X_is and computing $R(X_i)$.
  - Computing the limit ?

- $X_\infty = \cup\phi^n(X_0)$ is the strongest **invariant** of the program
- Looking for overapproximations : $X_\infty \subseteq X_{result}$ also called **invariant**.

Invariants for programs



- $\{x \in \mathbb{N}, 0 \leq x \leq 100\}$ is the most precise invariant in control point `loop`.

Back to our problem

Given a program (or an interpreted automaton), find inductive invariants for each control point : Recall : a **state** is a pair

(pc, Val) :

$$\mathrm{Val} : \mathrm{Var} \to \mathcal{N}^d$$

- We want to compute *lfp*(ϕ) with

$$\phi(X) = X_0 \cup \{y \in \Sigma \mid \exists x \in X \ x \to y\}$$

and → entails the **actions** of the program.

Representing sets of valuations

First problem to cope with : **represent sets of** valuations

$$\mathsf{Val} : \mathsf{Var} \to \mathcal{N}^d$$

- Var is $[\![0, \ldots, d-1]\!]$ (finite set, d vars)
- $\mathcal{N}$ is $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$

- Find a finite representation !

Computing R

Second problem to cope with : **computing** the transition relation

$$R(pc, X) = \{(pc', x') | \exists x \in X \text{ and } (pc, x) \rightarrow (pc', x')\}$$

- X is a (representation of a) set of valuations
- $\rightarrow$ is the program transition function.

- Let's try **intervals** (easy storage, easy computation) !

A first example

Try to compute an **interval** for each variable at each program point using **interval arithmetic** :

```
assume(x >= 0 && x<= 1);
assume(y >= 2 && y= 3);
assume(z >= 3 && z= 4);
t = (x+y) * z;
```

Interval for z? $[6, 16]$

Loops?

Push intervals / polyhedra forward...

```
int x=0;
while (x<1000) {
  x=x+1;
}
```

Loop iterations $[0,0]$, $[0,1]$, $[0,2]$, $[0,3]$,...

How? $\phi(X) = \mathrm{Initial\ state} \sqcup R(X)$, thus
$\phi([a,b]) = \{0\} \sqcup [a+1, \min(b, 999)+1]$

- Stricly growing interval during 1000 iterations, then stabilizes : $[0, 1000]$ is an **invariant**.

Termination Problem

Third problem to cope with : **stopping the computation** :

- Too many computations
- unbounded loops

One solution. . .

Extrapolation!

$[0,0]$, $[0,1]$, $[0,2]$, $[0,3] \to [0,+\infty)$

Push interval:

```
int x=0; /* [0, 0] */
while /* [0, +infty)*/ (x<1000) {
  /* [0, 999] */
  x=x+1;
  /* [1, 1000] */
}
```

Yes! $[0,\infty[$ is stable!

Computing inductive invariants as intervals

- Representation : intervals. The union leads to an overapproximation.
- We don't know how to compute $R(P)$ with P interval (The statements may be too complex, ...)
  ▶ Replace computation by simpler over-approximation $R(X) \subseteq R^\sharp(X)$.
- The convergence is ensured by **extrapolation/widening**.

▶ We always compute $\phi^\sharp(X)$ with : $\phi(X) \subseteq \phi^\sharp(X)$
In the end, **over-approximation** of the least fixed point of ϕ.

Computing inductive invariants as intervals - 2

Interval operations :

- $+, -, \times$ on intervals : interval arithmetic
- union : $[a, b] \cup [c, d]$: loosing info !
- **widening** : $(I_1 \nabla I_2$ with $I_1 \subseteq I_2)$

$$\bot \nabla I = I$$

$$[a, b] \nabla [c, d] = [\text{if } c < a \text{ then } -\infty \text{ else } a,$$

$$\text{if } d > b \text{ then } +\infty \text{ else } b]$$

The idea is to infer the dynamic of the intervals thanks to the first terms.

Computing inductive invariants as intervals - 3

The widening operator being designed, we compute ($x \subseteq F(x)$)

$$\Sigma_0, Y_1 = \Sigma_0 \nabla F(\Sigma_0), Y_2 = Y_1 \nabla F(Y_1) \ldots$$

finite computation instead of :

$$\Sigma_0, F(\Sigma_0), F^2(\Sigma_0), \ldots$$

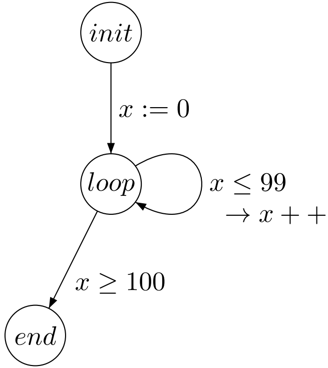
which can be infinite.

Theorem

(Cousot/Cousot 77) Iteratively computing the reachable states from the entry point with the interval operators and applying widening at entry nodes of loops converges in **a finite** *number of steps to a overapproximation of the least invariant (aka* **postfixpoint***).*

- The widening operators must satisfy the non ascending chain condition (see Cousot/Cousot 1977).

Invariants for programs - ex 1



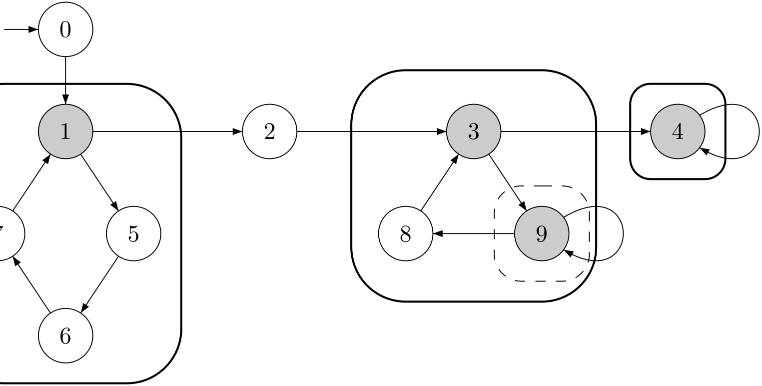
- $x \in [0, +\infty]$ in loop.

Computing inductive invariants as intervals - ex 2

```
x = random(0,7);
y = cos(x)+x
while (y<=100) {
  if (x>2) x--;
  else {
    y = -4;
    x--;
  }
}
```

Nested loops / Several loops

(Bourdoncle, 1992) Computing strongly connected subcomponents and iterate inside each :



Gray nodes are **widening nodes**

Improving precision after convergence - 1

```
int x=0; /* [0, 0] */
while /* [0, +infty)*/ (x<1000) {
  /* [0, 999] */
  x=x+1;
  /* [1, 1000] */
}
```

we got $[0, +\infty)$ instead of $[0, 999]$. Run one more iteration of the loop: $\{0\} \sqcup [1, 1000] = [0, 1000]$. Check if $[0, 1000]$ is an inductive invariant ? **YES**

- This is called **narrowing** or descending sequence : ends when we have an inductive invariant or after *k* applications of the transition function.

Improving precision after convergence - 2

Let $\hat{x}$ be the result of the computation

Result

The descending sequence always improves precision.

Proof : $lfp(F) \subseteq \hat{x}$, then $F(lfp(F)) = lfp(F) \subseteq F(\hat{x})$, and $F(\hat{x})$ is again a correct invariant. If $\hat{x}$ is not a fixpoint, then $F(\hat{x}) \subset \hat{x}$, so is a strictly better invariant.

Best invariant in domain not computable

```
P();
x=0;
```

Best invariant at end of program, as interval?

$[0,0]$ iff P() terminates
$\emptyset$ iff P() does not terminate

Entails solving the **halting problem**.

When intervals are not sufficient

```
assume(x >= 0 && x <= 1);
y = x;
z = x-y;
```

- The human (intelligent) sees $z = 0$ thus interval $[0, 0]$, taking into account $y = x$.
- Interval arithmetic does not see $z = 0$ because it does not take $y = x$ into account.

How to track relations

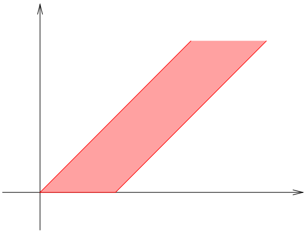
Using **relational domains**.

E.g.: keep

- for each variable an interval
- for each pair of variables (x, y) an information $x - y \leq C$.
- (One obtains $x = y$ by $x - y \leq 0$ and $y - x \leq 0$.)

How to **compute** on that?

Bounds on differences : practical example



Suppose $x - y \leq 4$, computation is $z = x + 3$, then we know $z - y \leq 7$.

Suppose $x - z \leq 20$, that $x - y \leq 4$ and that $y - z \leq 6$, then we know $x - z \leq 10$.

We know how to **compute** on these relations (transitive closure / shortest path).
On our example, obtain $z = 0$.

Why this is useful

Let $t(0..\mathbf{n})$ an array in the program.
The program writes $t(i)$.

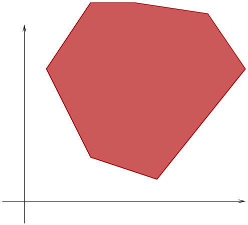
Need to know whether $0 \leq i \leq n$, otherwise said find bounds on i and on $n - i$...

Can we do better?

How about tracking relations such as $2x + 3y \leq 6$?

At a given program point, a set of **linear inequalities**.

In other words, a **convex polyhedron**.



Plan

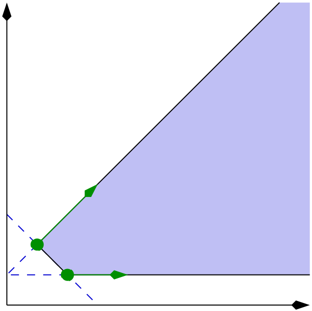
Intro

(Halbwachs/Cousot 1979)

  - Abstract Interpretation in the Polyhedral domain
  - Infinite Domain with many particularities
  - Discover affine relations on variables
- Classically used in verification problems.

The polyhedral domain (1)

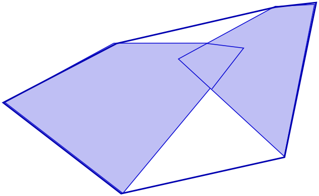
Convex polyhedra representation :



- Effective and efficient algorithmic (emptyness test, union, affine transformation . . .)

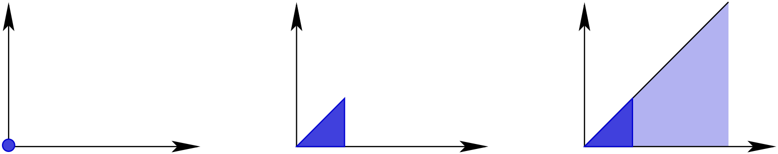
The polyhedral domain(2)

- Intersection, emptyness
- Affine Transformation : $a(P) = \{CX + D \mid X \in P\}$.
- Convex hull (loss of precision)



The Polyhedral domain (3)

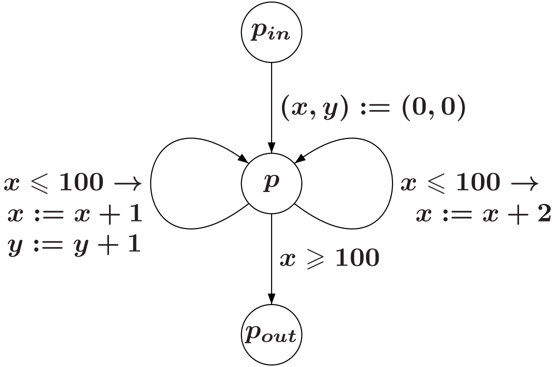
Widening : $P \nabla Q$: limit extrapolation.
$P \nabla Q$ constraints : take Q constraints and remove those which are not saturated by P.



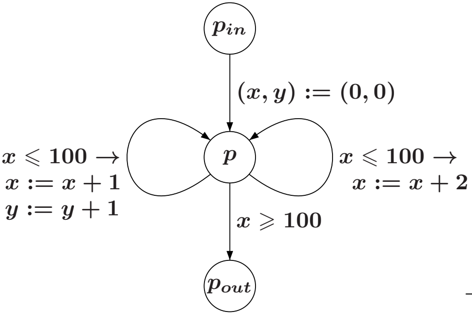
Trick (!) : $\{x = y = 0\} = \{0 \leq y \leq x \leq 0\}$

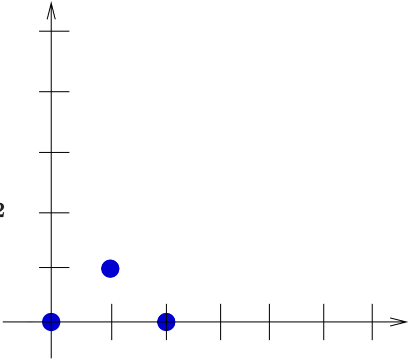
Analysis example - 1

```
x:=0;y:=0
while (x<=100) do
  read(b);
  if b then
          x:=x+2
        else begin
          x:=x+1;
          y:=y+1;
        end;
  endif
endwhile
```



Example - 2


p_{in}
$(x, y) := (0, 0)$
$x \leqslant 100 \rightarrow$
$x := x + 1$
$y := y + 1$
p
$x \leqslant 100 \rightarrow$
$x := x + 2$
$x \geqslant 100$
p_{out}



Linear Relation Analysis - Problems

Complexity increases with :

- number of control points
- number of numerical variables

Approximation is due to :

- Convex hulls
- Widening

(credits for these slides : Nicolas Halbwachs)

Complexity

(In general) The more precise we are, the higher the costs.

- Intervals: algorithms $O(n)$, n number of variables.
- Differences $x - y \leq C$: algorithms $O(n^3)$
- Octagons $\pm x \pm y \leq C$ (Miné) : algorithms $O(n^3)$
- Polyhedra (Cousot / Halbwachs): algorithms often $O(2^n)$.

Delaying widening - 1

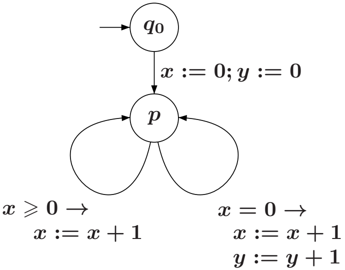
Halbwachs 1993 / Goubault 2001 / Blanchet et al. 2003

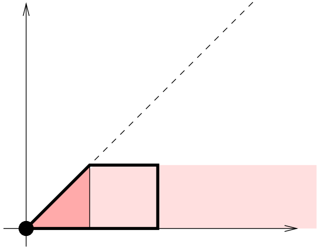
Fix k and compute :

$$X_n = \begin{cases} \bot & \text{if } n = 0 \\ F(X_{n-1}) & \text{if } n < k \\ X_{n-1} \nabla F(X_{n-1}) & \text{else.} \end{cases}$$

- Similar to unrolling loops, costly but useful (regular behaviour after a constant number of iterations).

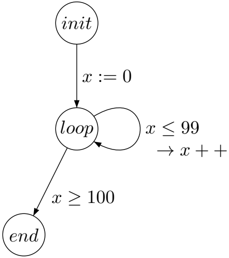
Delaying widening - 2 - ex





Improving the widening operator

While applying $P \nabla Q$, intersect with constraints that are satisfied by bith P and Q. The constraints must be precomputed.



Here, with "$x \leq 100$" in the pool of constraints, it avoids narrowing.

- Warning **widening is not monotone**, so improving locally is not necessarily a good idea !

Local improvement with acceleration

(Gonnord/Halbwachs 2006, Schrammel 2012)
Idea : Sometimes, a fixpoint of a loop can be easily computed without any fixpoint iteration.

More details here

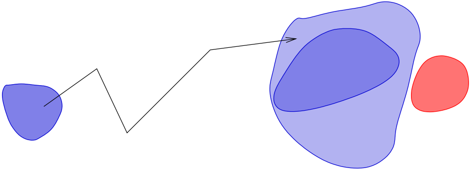
Good path heuristic

(Gonnord/Monniaux 2011)
Idea : find interesting paths by means of smt-queries

More details here

Applications

- Bounds on iterators of arrays (intervals, differences on bounds)
- Dead code elimination (all domains) - especially when the code has been automatically generated / asserts
- Vectorization : computations that can be permuted
- Memory optimisation : this `int` can be encoded in 16 bits ?
- Preconditions for code specialization (on going work with F. Rastello)
- Safety analysis



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Tools - Academic

ASPIC : Accelerated Symbolic Polyhedral Invariant Computation

Aspic is **an invariant generator** :

- From counter automata with numerical variables.
- Invariants are **polyhedra**.

C2fsm is **a C parser** :

- From a source file in (a subset of) C into Aspic input language (fast).
- **Safe** abstractions of non numerical variables, structures, behaviors.

- `http://laure.gonnord.org/pro/aspic/aspic.html`

Tools - More robust

- Frama-C : analysing/ proving correction of C programs (see `http://frama-c.com/`
- Apron : numerical domain interface (`http://apron.cri.ensmp.fr/library/`)
- Interproc : IA analyser connected to Apron (see `http://pop-art.inrialpes.fr/interproc/interprocweb.cgi`
- Rose / LLVM : C (and more) parsers and API for manipulating C programs. Rose is more decidated to program transformation, LLVM to compiler construction(`http://www.rosecompiler.org/` and `http://llvm.org/`.

Industrial succes stories

  - Polyspace
  - Astree
- See later for anecdotes.

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LRA and acceleration

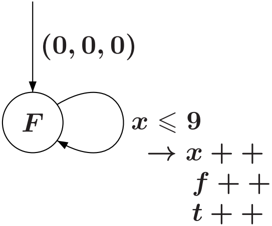
(Gonnord/Halbwachs 2006, Gonnord/Schrammel 2012, Schrammel/Jeannet 2014)

Combination LRA and acceleration techniques [Finkel/Sutre/Leroux/...]

- Abstract acceleration notion :
  - low-cost overapproximations;
  - inside LRA, combination with widening.
- Classification of accelerable loops.
- Prototype : ASPIC

Acceleration - accelerable loops

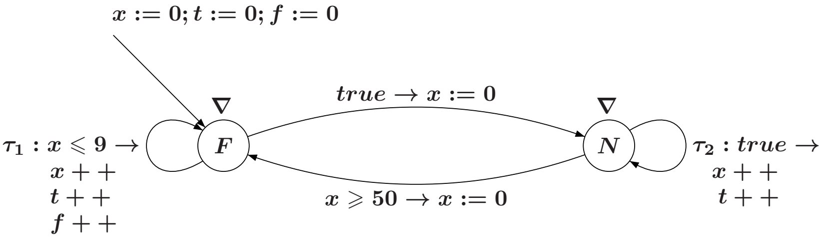
An easy case



- **exact effect** : $\exists i \in \mathbb{N}, x = f = t = i, 0 \leq i \leq 10$
- **exact effect in the abstract domain** :

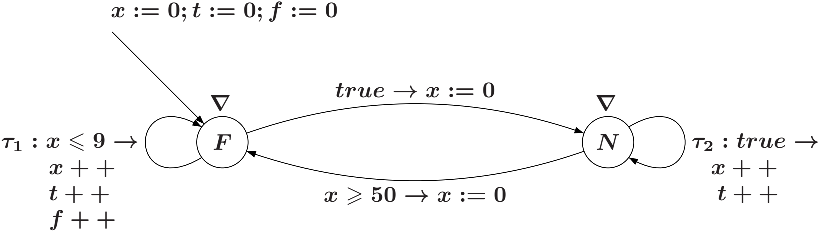
$\{x = f = t, 0 \leq t \leq 10\}$

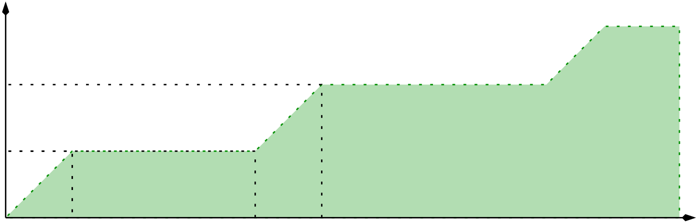
Gas Burner example - 1



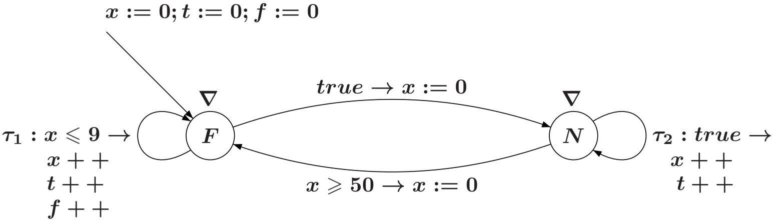
- f global leaking time
- t global time
- x local variable

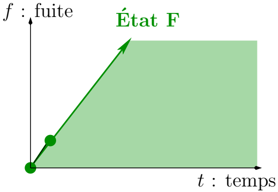
Gas burner 2 - Real Behaviour

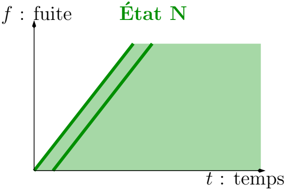




Gas burner 3 - with LRA

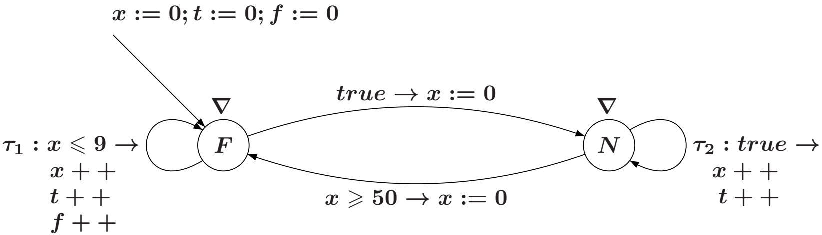


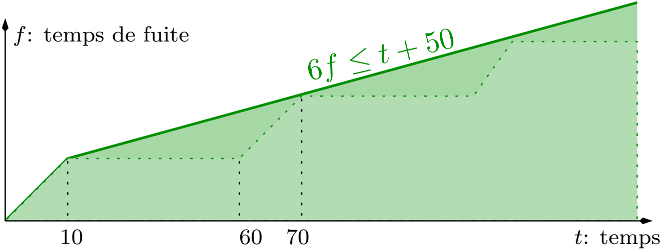




- Loss of precision

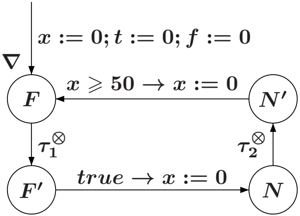
Gas burner - desired invariant





Accelerating the gas burner - 1

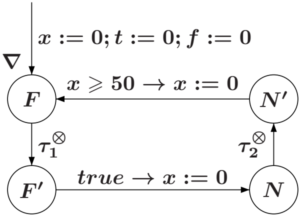
(mini-)loops are replaced ($\tau_i : g_i \rightarrow a_i$)



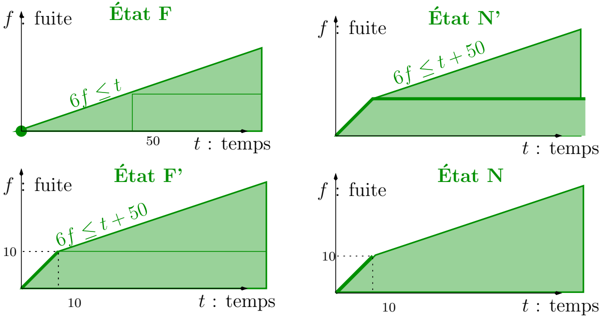
- $\tau_i^{\otimes}$ summarizes the effect of any application of τ_i (unfixed number of iterations).
- Outer loop is **widened**.

Accelerated Gas Burner - 2

back



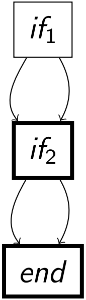
- $\tau_1^{\otimes}$ = "add ray $(1, 1, 1)$ while $x \leq 10$"
- $\tau_2^{\otimes}$ = "add ray $(1, 0, 1)$"



SMT+LRA, Motivation : example 1

Some properties cannot be expressed in convex abstract domains:

```
if (x >= 0) { xabs = x; }
  else { xabs = -x; }
if (xabs >= 0.01) {
  y = (sin(x) / x) - 1;
} else {
  xsq = x*x;
y = xsq*(-1/6. + xsq/120.);
}
```

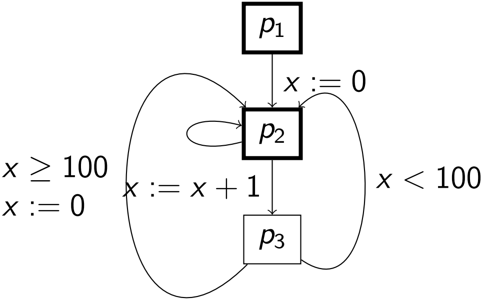


- Store the fact that $x = xabs \vee x = -xabs$ at node $if2$

SMT+ LRA, Motivation : example 2

The widening operator can be too coarse:

```
int x = 0;
while (true) {
  if (nondet()) {
    x = x+1;
    if (x >= 100) x = 0;
} }
```



- The analysis (interval domain) gives $[0, +\infty)$, not improved by narrowing !

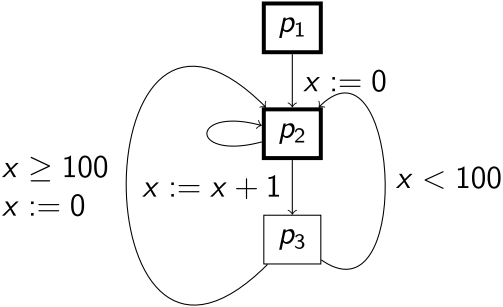
Two ideas

- First idea : do not compute the convex hull on “diamonds”.
- Second idea : consider all paths and analyse them **separately**.

- Advantage : precision
- Drawback : combinatorial explosion

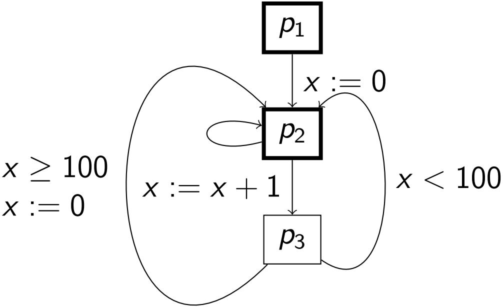
▶ Our method will implement these two ideas without computing all the program paths explicitely.

Invariants



Is $x = \mathbf{0}$ an invariant in p_2 ?

Invariants

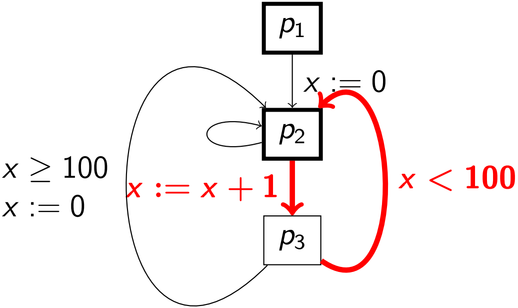


Is $x = \mathbf{0}$ an invariant in p_2 ?

- No ! Because it's not stable.

Our method on example 2

Focus on the red path !



- Its least inductive invariant (for p_2) is $x \in [0, 99]$, which is also an invariant while considering the whole graph.

How to detect paths ? back

We delegate the search for new paths to an SMT solver.
The problem is encoded into an SMT-problem thanks to the use of an internal **structure**:

- compact; (complexity reasons)
- acyclic; (to reason about loops as for paths)
- all variables are assigned once (to reason about unique variable values).

- Preprocessing : computing this structure.

(SAS 2011 for technical details)