Informatique Fondamentale IMA S8

Cours 2 - Stack automata + Grammars

Laure Gonnord

http://laure.gonnord.org/pro/teaching/

Laure.Gonnord@polytech-lille.fr

Université Lille 1 - Polytech Lille

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II - Stack automata and Grammars

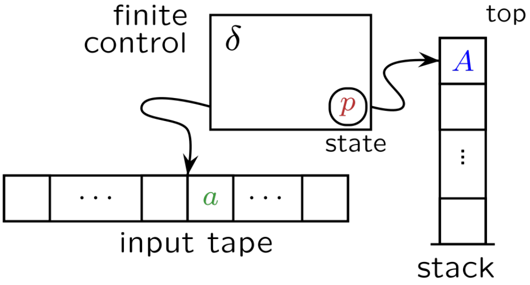
What for ?

Express more than regular languages !

- Give a way for automata to “**count**”, to “have a memory”.

Example

(source : Wikipedia)



General definition

Stack/P automata

- States (Q), initial state (q_0), finite states (F).
- Two alphabets (one for read : Σ one for stack : Γ)
- $\gamma_0 \in \Gamma$ the end of stack character.
- Transitions are finitely many.
- A **stack** to write into.
- Transitions use the stack.

▶ The transition function is :

$$Q \times (\Sigma \cup \varepsilon) \times \Gamma \to \mathcal{P}(Q \times \Gamma^*)$$

How it works ? 1/3

Configuration

A **configuration** is a triple (q, w, α) where :

- q is the current state
- $w \in \Sigma^*$ the part of the input tape which is not yet read
- $\alpha \in \Gamma^*$ the current stack word

Two different conditions for accepting words :

- "**accepting state**" : the read word leads to a configuration of the form (q, ε, α) where $q \in F$ (with any α)
- "**empty stack**" : ... $(q, \varepsilon, \gamma_0)$ (with any q, but γ_0 is the "empty stack" character.

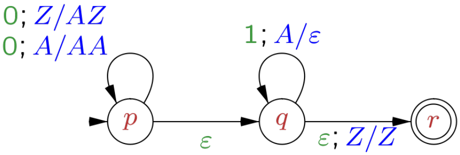
How it works ? 2/3

Given a configuration (q, w, α), the next one is computed by the help of the transition function $Q \times (\Sigma \cup \varepsilon) \times \Gamma \to \mathcal{P}(Q \times \Sigma^*)$:

- **enabled transitions** : those of the form $(q, a, A) \to (q', A')$ where a is the next letter to read on the input tape (or ε), and A the letter on top of the stack : $w = aw'$ and $\alpha = A\alpha'$.
- Pick one enabled transition, and compute the next configuration $(q', w', A'\alpha')$.

How it works ? 3/3

(source : wikipedia)



- Try to derive 0011 and 00111 !
- The PDA recognises $\{0^n 1^n | n \in \mathbb{N}\}$ by accepting state.

Important theoretical results on PDA

- Important Stack automata are non deterministic !
- The two acceptance criteria define the same class of languages

Goal

Problem : Express languages with the same expressivity as stack automata.

- use **grammars**

General grammars

Grammar rule

A **grammar** rule (production rule) is of the form

$$w \longrightarrow w'$$

where w and w' are words.

A grammar is a set of rules.

Grammars

Grammar

A **grammar** is composed of :

- A finite set N of non terminal symbols
- A finite set Σ of terminal symbols (disjoint from N)
- A finite set of production rules, each rule of the form $w \to w'$ where w is a word on $\Sigma \cup N$ with at least one letter of N. w' is a word on $\Sigma \cup N$.
- A start symbol $S \in N$.

Grammars

Example :

$$S \to aSb$$

$$S \to \varepsilon$$

is a grammar with $N =$ .... and ....

Associated Language

Derivation

G a grammar defines the relation :

$$x \Rightarrow_G y \text{ iff } \exists u, v, p, q x = upv \text{ and } y = uqv \text{ and } (p \to q) \in P$$

- A grammar describes a **language** (the set of words on Σ that can be derived from the start symbol).

Examples

$$S \to aSb$$

$$S \to \varepsilon$$

The grammar defines the language $\{a^n b^n, n \in \mathbb{N}\}$

$$S \to aBSc$$

$$S \to abc$$

$$Ba \to aB$$

$$Bb \to bb$$

The grammar defines the language $\{a^n b^n c^n, n \in \mathbb{N}\}$

- Exercises

Context-free grammars

Context-free grammar

A **CF-grammar** is a grammar where all production rules are of the form $N \rightarrow (\Sigma \cup N)^*$.

Example of CF-grammar

$$S \to S + S | S * S | a$$

The grammar defines a language of arithmetical expressions.

- Notion of **derivation tree**.

Draw a derivation tree of a*a+a, of S+S !

Relationship between stack automata and grammars

General result

The context-free/algebraic languages are exactly the languages that are described by stack automata.

- The proof is not difficult.
- Exercises

Some other results/definitions

- Regular languages are algebraic languages, but not the converse.
- There exists normal forms for algebraic grammars
- A grammar can be ambiguous.

Summary

Stack Automata or **PushDown Automata** are :

- Acceptors for context-free languages. But some languages are not context free.
- Non deterministic.

▶ http://en.wikipedia.org/wiki/Pushdown_automaton for useful pointers