Informatique Fondamentale IMA S8

Cours 1 - Intro + schedule + finite state machines

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Until now

During S5,S6,S7, IMA students have learnt :

- (programming stuff) C programing and compiling,
- (conception stuff) algorithm design and encoding,
- (microelec) circuits and embedded systems hardware,
- (auto) design flows of industrial processes

- Solved (computation) *problems* by ad-hoc solutions.

Until now 2/2

But :

- No general scheme to design algorithms.
- (manual) evaluation of the *cost* of our programs.
- worse no assurance of correctness.
- even worse it there always a solution ?

▶ All these problems will be addressed in this course

Schedule

- Finite state machines (regular automata), regular languages. Notion of non determinism. Link with circuits.
- I/O automata, stack automata and grammars. Link to “simple” languages.
- Counter automata, Turing machines and undecidable problems. Link to “classical” programs.
- Graphs and classical problems/algorithms on graphs.
- Compiler Construction. (+ lab)

I - Regular languages and automata

What for ?

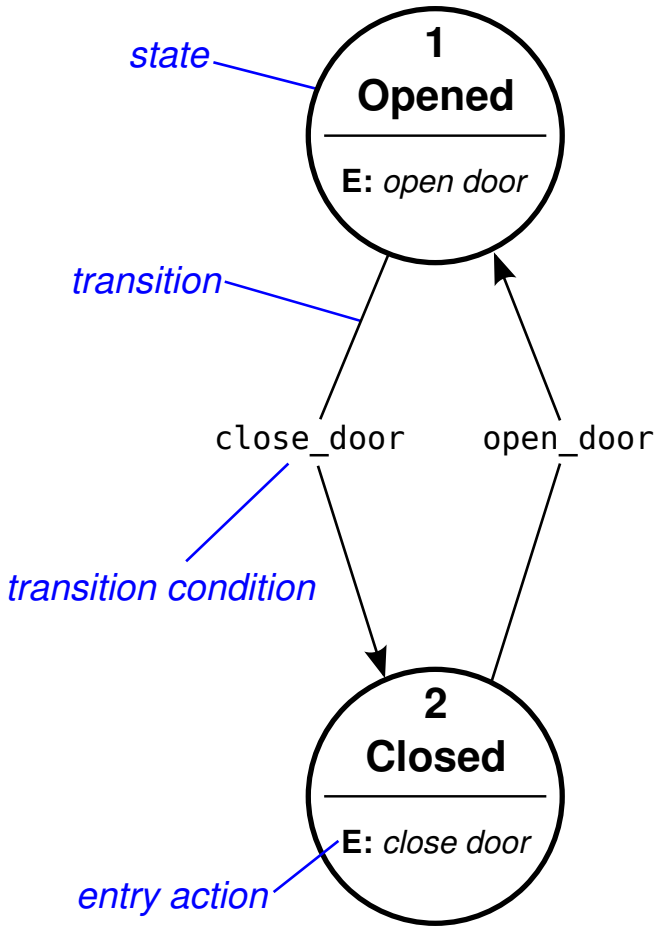
We want to model :

- the behaviour of systems with behavioural **modes**.
- the behaviour of (Boolean) circuits.
- sets of words.

- A finite representation.

Example

Opening/Closing door - Source Wikipedia.

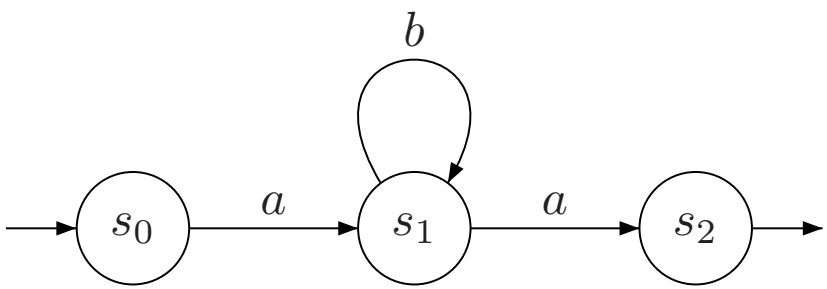


General definition

Finite state machine (FSM) or **regular automata**

- States are labeled, and there are finitely many ($s \in Q$)
- Initial state(s) ($i \in I$) and accepting (terminating/finishing) states ($t \in F$)
- Transitions are finitely many.
- Transitions are labeled with **letters** ($a \in A$).

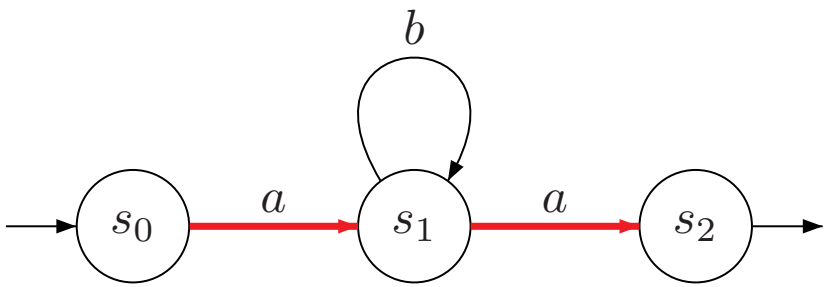
▶ The transition function is $\delta : Q \times A \to Q$.



Accepted language 1/2

Accepted word

A **word** w on the alphabet A is **accepted** by the automaton iff there exists a **finite** path from an initial state to an accepting state which is labeled by w.



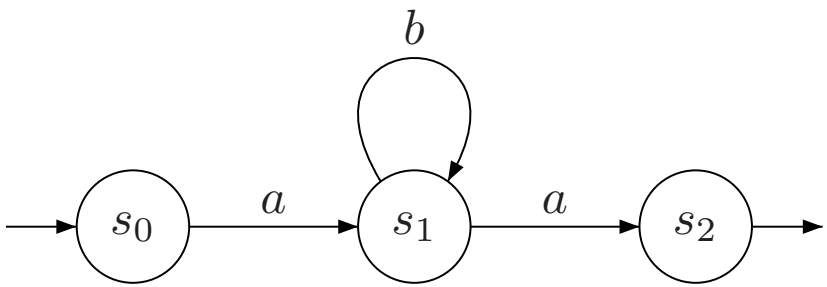
- $w = aa$ is accepted/recognised.

Accepted language 2/2

Accepted language

The **accepted language** of a given automaton $\mathcal{A}$ is the set of all accepted words and is denoted by $\mathcal{L}(\mathcal{A})$.

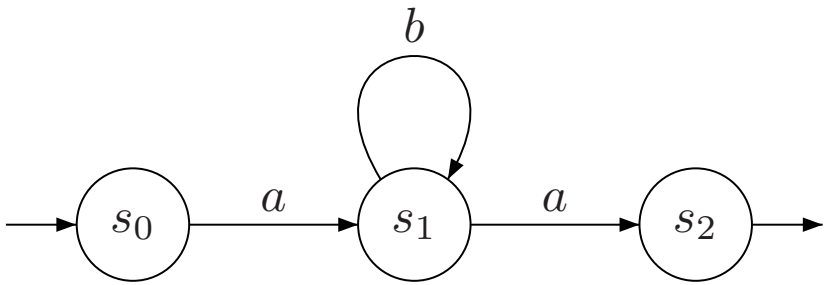
- Remark : it can be infinite.



- $\mathcal{L}(\mathcal{A}) = \{ab^k a, k \in \mathbb{N}\}$.

Data Structure for implementation

Problem : how to **encode** an automata ?



The transition function can be (for instance) encoded as a **transition table** :

	s_0	s_1	s_2
s_0	$--$	a	$--$
s_1	$--$	b	a
s_2	$--$	$--$	$--$

Goal

Problem : how to describe languages easily in a textual “linear” way ?

- use **regular expressions**.

Regular expressions

Regular expression (recursive def)

A **regular expression** e on the alphabet A is defined by induction. It can be of any of the following kinds :

- the empty word ε
- a letter $a \in A$
- a choice between an expression e_1 and another expression e_2 : $e_1 + e_2$
- two successive expressions : $e_1 \cdot e_2$
- 0,1 or more successive occurrences of e_1 : e_1^*.

Example with $A = \{a, b, c, d\}$: $e = a \cdot (b + c \cdot d)^*$

Regular expression vs word

A regular expression “encodes” the form of a word. For instance :

$$i \cdot m \cdot a \cdot (3 + 4 + 5)$$

(on the alphabet $\{i, m, a, 3, 4, 5\}$) describes all words beginning by the prefix “ima” and finishing by one of the numbers $3, 4$ or 5.

- A regular expression describes a **language**

Regular language

Regular language

Given a regular expression e, $\mathcal{L}(e)$ denotes the set of words (the **language**) that are described by the regular expression e.

Example with $A = \{0, \ldots, 9\}$:
$e = (1 + 2 + \ldots + 9) \cdot (0 + 1 + 2 + \ldots + 9)^*$. What is $\mathcal{L}(e)$?

Linux world

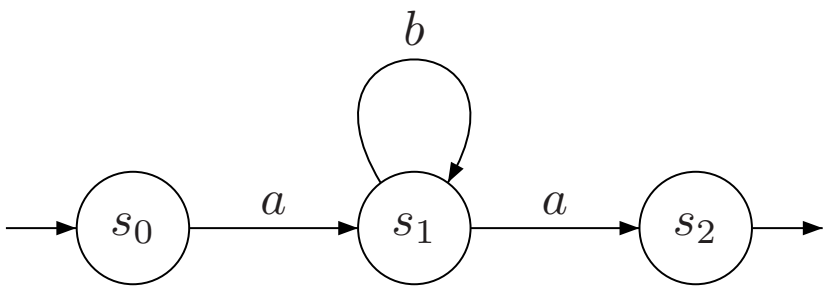
Some commands use (extended) regular expressions (regexp) :

- `ls *.pdf` lists all pdfs of the current directory
- `grep ta*.* *.c` find all lines in .c files that contains words that begin with t + some a's.
- `sed ''s ta*.*/toto/g'' file` replace all occurrences...

Relationship between automata and languages - 1/3

First, some experiments.

Given the following automata $\mathcal{A}$, are you able to give a regular expression e such that $\mathcal{L}(e) = \mathcal{L}(\mathcal{A})$?



- $e =?$

Relationship between automata and languages - 2/3

And the converse :

Given the following regular expression $e = b^* \cdot (c + a)^*$, are you able to give an automaton $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) = \mathcal{L}(e)$?

Relationship between automata and languages - 3/3

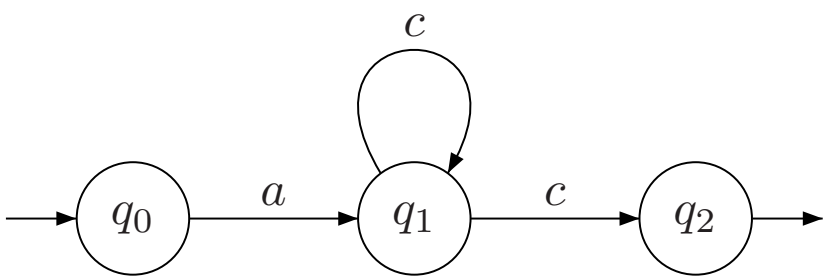
General result - Kleene Theorem

The regular languages are exactly the languages that are described by finite automata.

- What we've done before is always possible.

Goal

Sometimes some info lacks to make a choice between two transitions :



- From state q_1, there is a **non deterministic choice** while reading c : either go to state q_2 or stay in q_1.

Definition

Non deterministic FSM

- A deterministic automaton is $\mathcal{A} =< A, Q, I, F, \delta >$ with

$$\delta : A \times Q \to Q$$

- A **non deterministic** automata is the same with :

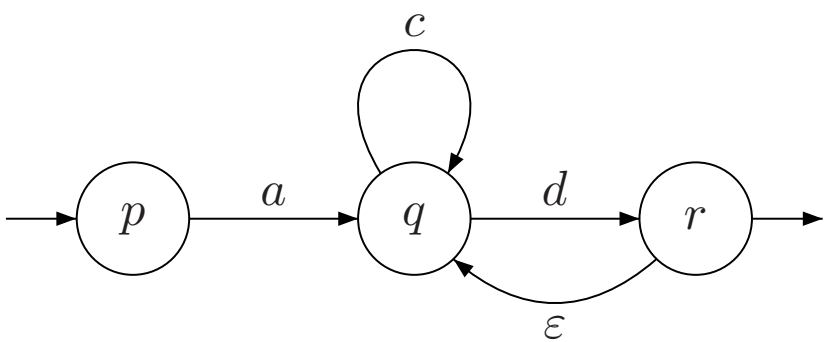
$$\delta : A \times Q \to P(Q)$$

- A **non deterministic** automata with ε-transitions is the same with

$$\delta : (A \cup \{\varepsilon\}) \times A \to P(Q)$$

Example

A non deterministic automata with ε-transitions :



- Then $\mathcal{L}(\mathcal{A}) = a \cdot (c^* \cdot d)^*$.

Find the associated language

Goal : Given $\mathcal{A}$ an automaton, find the associated language.

- Exercises !

Construction of automaton from a regular expression

Goal : Given a regular expression, construct a regular automaton that "**recognises**" it.

- Exercises !

Determinisation

Goal : transform a non deterministic automaton into a deterministic one.

- Exercises !

Other Algorithms

In the literature you will easily find :

- algorithms to eliminate ε transitions without determinising (ε closure) ;
- algorithms to minimise automata (the number of states) ;
- algorithms to use automata to find words in a text ;
- algorithms to test language inclusion (if they are regular)
- ...

Non regular languages

Important result

There exists some non-regular languages.

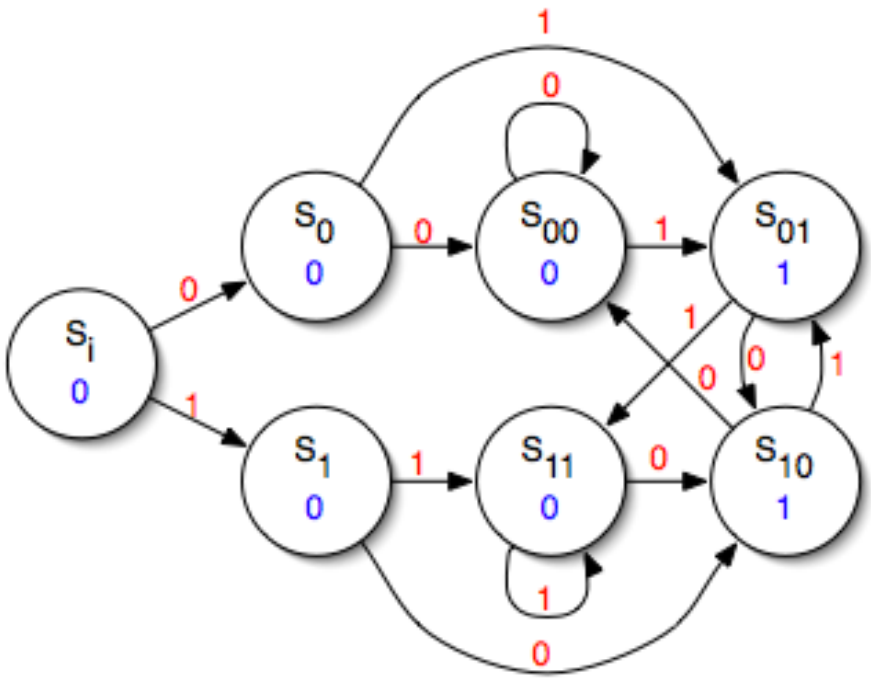
Examples of non-regular languages :

- $\{a^n b^n, n \in N\}$.
- palindromes on a non-singleton alphabet.
- $\{a^p, p \text{ prime}\}$

▶ There exists a quite systematic way to prove that a given language is not regular (Pumping Lemma).

Moore and Mealy Machines - 1

Moore : I/O machine whose output values are determined solely by the current state :



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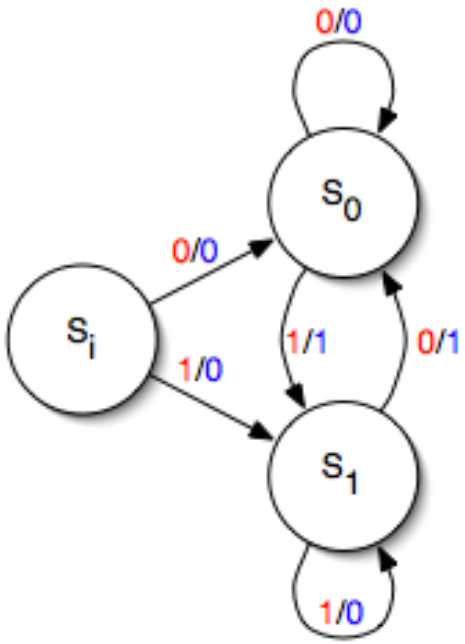
Moore and Mealy Machines - 2

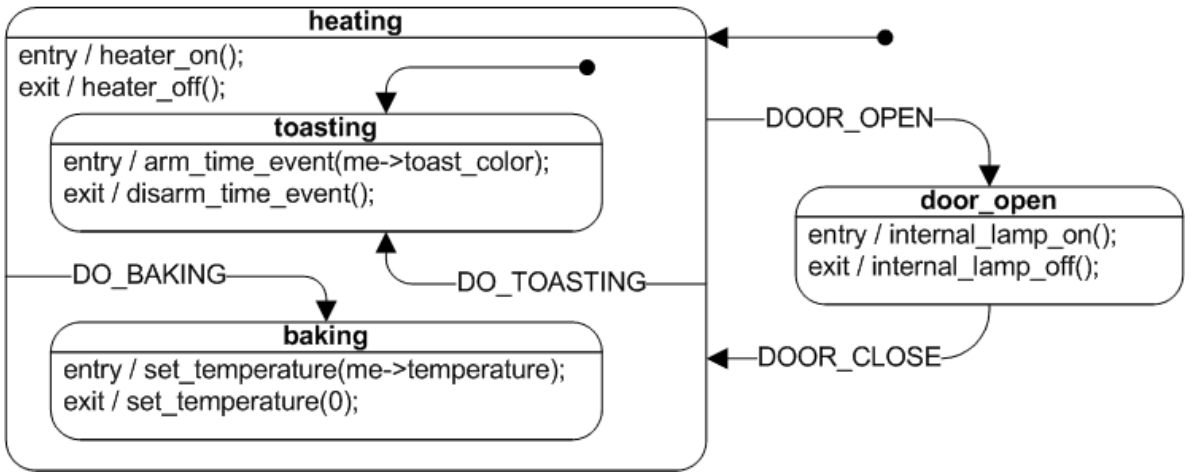
Mealy : output values are determined both by the current state and the value of the input.



source Wikipedia

UML Implementation of FSMs

UML variants of FSMs are **hierarchical**, react to messages and call functions.



source Wikipedia

Hardware Implementation of FSMs

It requires :

- a register to store state variables
- a block of combinational logic for the state transition
- (optional) a block of combinatorial logic for the output

Summary

Regular Automata or **Finite State Machines** are :

- Acceptors for regular languages. But some languages are not regular.
- Algorithmically efficient.
- Useful to describe (simple) behaviours of systems.
- Closely linked to circuits.