

# Static Analysis

## Master Course in ÉNS LYON

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# Context

Before actually generating code / allocate registers, it would be useful to get some information on

- Variables : value range, scope, lifespan, constants, . . .
  - Arrays : illicit accesses, alias discovery. . .
  - Data Structures : memory leaks, null pointer dereferences. . .
- ▶ static analyses, of different kinds

- 1 Data Flow analysis
  - Available expressions
  - Live Variable analysis
  - Toward a generalisation of these analysis
- 2 Abstract Interpretation
- 3 Linear Relation Analysis
- 4 And ?

# What for ?

Avoiding the computation of an (arithmetic) expression :

```
x:=a+b;  
y:=a*b;  
while(y>a+b) do  
    a:=a+a;  
    x:=a+b;  
done
```

## Some defs

### Definition

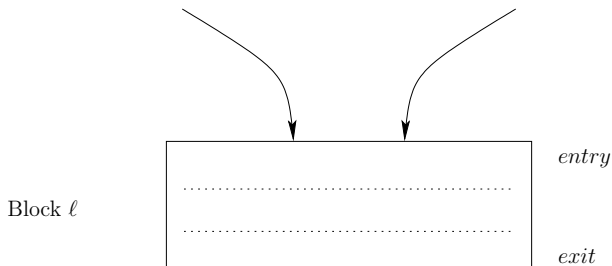
An expression is **killed** in a block if any of its variables is used in the block.

### Definition

A **generated** expression is an expression evaluated in the block and none of its variables is killed in the block.

► Sets :  $kill_{AE}(block)$  and  $gen_{AE}(block)$

# Data flow expressions

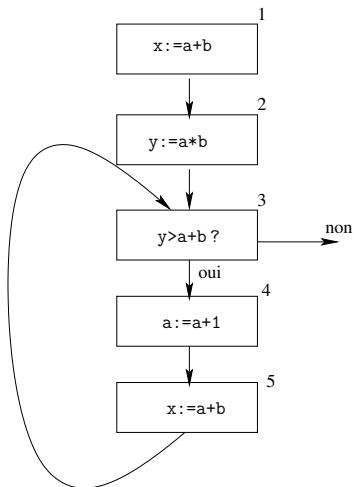


$$AE_{entry}(\ell) = \begin{cases} \emptyset & \text{if } \ell = \textit{init} \\ \bigcap \{AE_{exit}(\ell') \mid (\ell', \ell) \in \textit{flow}(G)\} & \end{cases}$$

$$AE_{exit}(\ell) = (AE_{entry}(\ell) \setminus \textit{kill}_{AE}(\ell)) \cup \textit{gen}_{AE}(\ell)$$

# On the example - equations

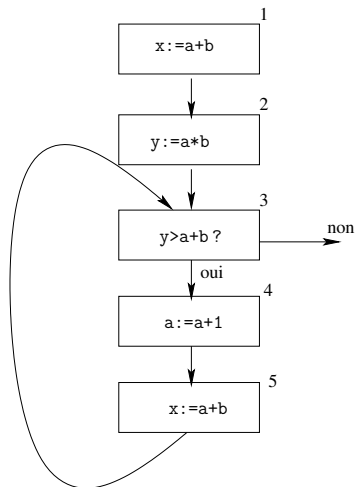
$\ell$	$kill_{AE}(\ell)$	$kill_{AE}(\ell)$
1	$\emptyset$	$\{a+b\}$
2	$\emptyset$	$\{a*b\}$
3	$\emptyset$	$\{a+b\}$
4	$\{a+b, a*b, a+1\}$	$\emptyset$
5	$\emptyset$	$\{a+b\}$



# On the example - final solution

$\ell$	$AE_{entry}(\ell)$	$AE_{exit}(\ell)$
1	$\emptyset$	$\{a+b\}$
2	$\{a+b\}$	$\{a*b, a*b\}$
3	$\{a+b\}$	$\{a+b\}$
4	$\{a+b\}$	$\emptyset$
5	$\emptyset$	$\{a+b\}$

- ▶  $a+b$  is available on entry to the loop, not  $a*b$
- ▶ Improvement of code generation



## Another example : live ranges

```
x:=2;  
y:=4;  
x:=1;  
if (y>x) then z:=y else z=y*y ;  
x:=z;
```

### Definition

A variable is **live** at the exit of a block if there exists a path from the block to a use of the variable that does not redefine the variable.

Problem : determine the set of variables that *may be* live after each control point.

# Data flow expressions

## Definition

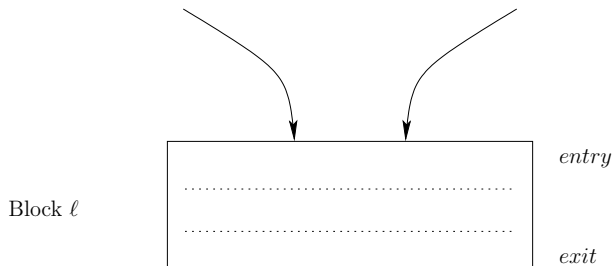
A variable that appears on the left hand side of an assignment is **killed** by the block. Tests do not kill variables.

## Definition

A **generated** variable is a variable that appears in the block.

► Sets :  $kill_{LV}(block)$  and  $gen_{LV}(block)$

# Data flow expressions



$$LV_{exit}(\ell) = \begin{cases} \emptyset & \text{if } \ell = final \\ \bigcup \{LV_{entry}(\ell') \mid (\ell', \ell) \in flow(G)\} & \end{cases}$$

$$LV_{entry}(\ell) = (LV_{exit}(\ell) \setminus kill_{LV}(\ell)) \cup gen_{LV}(\ell)$$

## Final result and use

**Backward** analysis and we want the smallest sets, here is the final result : (we assume all vars are dead at the end).

$\ell$	$LV_{entry}(\ell)$	$LV_{exit}(\ell)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
5	$\{y\}$	$\{z\}$
5	$\{y\}$	$\{z\}$
5	$\{z\}$	$\emptyset$

► Use : Dead code elimination ! Note : can be improved by computing the use-defs paths. (see Nielson/Nielson/Hankin)

## Common points

- Computing growing sets from  $\emptyset$  via *fixpoint iterations*. (or the dual)
- Sets of equations of the form (collecting semantics) :

$$\mathcal{S}(\ell) = \bigcup_{(\ell', \ell) \in E} f(\mathcal{S}(\ell'))$$

where  $f$  is computed w.r.t. the *program statements*

- $\mathcal{S}$  is an **abstract interpretation** of the program.

- 1 Data Flow analysis
- 2 Abstract Interpretation
  - Transition systems and invariants
  - Computing Invariants (forward)
  - Non-relational vs relational analyses
- 3 Linear Relation Analysis
- 4 And ?

# Goal

Propagating **information** about program variables (numerical, arrays, ...) in order to get **invariants**.

▶ We focus on **numerical variables** here.

Credits for slides : David Monniaux

## Initial states + transitions

Program or machine state = values of variables, registers, memories. . . within state space  $\Sigma$ .

Examples :

- if system state = 17-bit value, then  $\Sigma = \{0, 1\}^{17}$  ;
- = 3 unbounded integers,  $\Sigma = \mathbb{Z}^3$  ;
- if finite automaton,  $\Sigma$  is the set of states ;
- if stack automaton, complete state = couple (finite state, stack contents), thus  $\Sigma = \Sigma_S \times \Sigma_P^*$ .

**Transition relation**  $\rightarrow x \rightarrow y$  = “if at  $x$  then can go to  $y$  at next time”

## Reachable states

Let  $\Sigma_0 \subseteq \Sigma$  the set of initial states of the program.

The **reachable** states are obtained by successively applying the transition relation, hence  $\sigma$  is reachable iff :

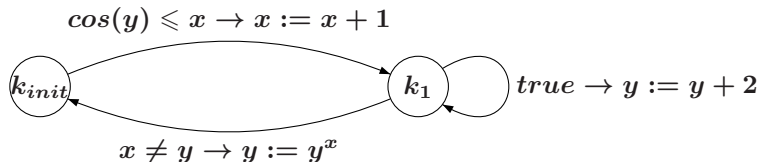
$$\exists \sigma_0 \in \Sigma_0 \sigma_0 \rightarrow^* \sigma$$

We also define  $X_n$  as the set of states reachable in at most  $n$  turns :  $X_0 = \Sigma_0$ ,  $X_1 = \Sigma_0 \cup R(\Sigma_0)$ ,  $X_2 = \Sigma_0 \cup R(\Sigma_0) \cup R(R(\Sigma_0))$ , etc.

with  $R(X) = \{y \in \Sigma \mid \exists x \in X \ x \rightarrow y\}$ .

The sequence  $X_k$  is ascending for  $\subseteq$ . Its limit (= the union of all iterates) is the **set of reachable states**.

# Reachable states for programs



Semantics of the programs as **transition systems** :

- A **state** is a couple  $(pc, Val)$  :

$$Val : Var \rightarrow \mathcal{N}^d$$

- $Var$  is  $\llbracket 0, \dots, d - 1 \rrbracket$  (finite set,  $d$  vars)
- $\mathcal{N}$  is  $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$
- **Initial** states :  $(pc_0, allv)$ .

# Iterative computation

Remark  $X_{n+1} = \phi(X_n)$  with  $\phi(X) = \Sigma_0 \cup R(X)$ .

How to **compute efficiently** the  $X_n$  ? And the limit ?

- Explicit representations of  $X_n$  (list all states) : If  $\Sigma$  finite,  $X_n$  converges in at most  $|\Sigma|$  iterations.
- else, we have to cope with two problems :
  - Representing the  $X_i$ s and computing  $R(X_i)$ .
  - Computing the limit ?

# Inductive invariants

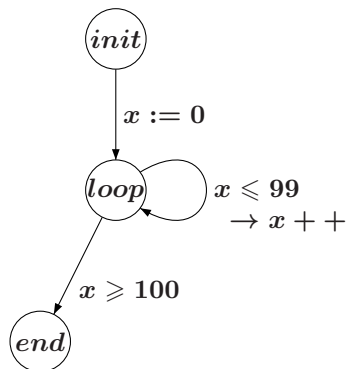
(Inductive) invariant : set  $X$  of states s.t.  $\phi(X) \subseteq X$  : with

$$\phi(X) = X_0 \cup \{y \in \Sigma \mid \exists x \in X \ x \rightarrow y\}$$

Properties :

- If  $X$  et  $Y$  two invariants, then so is  $X \cap Y$ .
  - $\phi$  **monotonic** for  $\subseteq$  (if  $X \subseteq Y$ , then  $\phi(X) \subseteq \phi(Y)$ ).
  - $\phi(X \cap Y) \subseteq \phi(X) \subseteq X$ , same for  $Y$ , thus  $\phi(X \cap Y) \subseteq X \cap Y$ .
  - Same for intersections of infinitely many invariants.
- Thus the **strongest invariant** can be defined as the intersection of all invariants. This invariant satisfies  $\phi(X) = X$ , it is the **least fixed point** of  $\phi$ .

# Invariants for programs



- ▶  $\{x \in \mathbb{N}, 0 \leq x \leq 100\}$  is the most precise invariant in control point `loop`.

## Back to our problem

Given a program (or an interpreted automaton), find inductive invariants for each control point : Recall : a **state** is a couple

$(pc, \text{Val})$  :

$$\text{Val} : \text{Var} \rightarrow \mathcal{N}^d$$

► We want to compute  $lfp(\phi)$  with

$$\phi(X) = X_0 \cup \{y \in \Sigma \mid \exists x \in X \ x \rightarrow y\}$$

and  $\rightarrow$  entails the **actions** of the program.

# Representing sets of valuations

First problem to cope with : **represent sets of** valuations

$$\text{Val} : \text{Var} \rightarrow \mathcal{N}^d$$

- $\text{Var}$  is  $\llbracket 0, \dots, d - 1 \rrbracket$  (finite set,  $d$  vars)
  - $\mathcal{N}$  is  $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$
- ▶ Find a finite representation !

# Computing R

Second problem to cope with : **computing** the transition relation

$$R(pc, X) = \{(pc', x') \mid \exists x \in X \text{ and } (pc, x) \rightarrow (pc', x')\}$$

- $X$  is a (representation of a) set of valuations
  - $\rightarrow$  is the program transition function.
- ▶ Let's try **intervals** (easy storage, easy computation) !

## A first example

Try to compute an **interval** for each variable at each program point using **interval arithmetic** :

```
assume(x >= 0 && x <= 1);
```

```
assume(y >= 2 && y = 3);
```

```
assume(z >= 3 && z = 4);
```

```
t = (x+y) * z;
```

Interval for  $z$  ? **[6, 16]**

# Loops ?

Push intervals / polyhedra forward. . .

```
int x=0;
while (x<1000) {
  x=x+1;
}
```

Loop iterations  $[0, 0]$ ,  $[0, 1]$ ,  $[0, 2]$ ,  $[0, 3]$ , . . .

How ?  $\phi(X) = \text{Initial state} \sqcup \text{post}(X)$ , thus  
 $\phi([a, b]) = \{0\} \sqcup [a + 1, \min(b, 999) + 1]$

► Stricly growing interval during 1000 iterations, then stabilizes :  $[0, 1000]$  is an inductive invariant.

# Termination Problem

Third problem to cope with : **stopping the computation** :

- Too many computations
- Non bounded loops

# One solution...

## Extrapolation !

$[0, 0], [0, 1], [0, 2], [0, 3] \rightarrow [0, +\infty)$

Push interval :

```
int x=0; /* [0, 0] */
while /* [0, +infty) */ (x<1000) {
  /* [0, 999] */
  x=x+1;
  /* [1, 1000] */
}
```

Yes !  $[0, \infty[$  is stable !

# Computing inductive invariants as intervals

- Representation : intervals. The union leads to an overapproximation.
- We don't know how to compute  $R(P)$  with  $P$  interval (The statements may be too complex, ...)
  - ▶ Replace computation by simpler over-approximation  $R(X) \subseteq R^\sharp(X)$ .
- The convergence is ensured by **extrapolation/widening**.
- ▶ We always compute  $\phi^\sharp(X)$  with :  $\phi(X) \subseteq \phi^\sharp(X)$   
In the end, **over-approximation** of the least fixed point of  $\phi$ .

## Computing inductive invariants as intervals - 2

Interval operations :

- $+$ ,  $-$ ,  $\times$  on intervals : interval arithmetic
- union :  $[a, b] \cup [c, d]$  : loosing info !
- **widening** :  $(I_1 \nabla I_2$  with  $I_1 \subseteq I_2$ )

$$\perp \nabla I = I$$

$$[a, b] \nabla [c, d] = [\text{if } c < a \text{ then } -\infty \text{ else } a, \text{if } d > b \text{ then } +\infty \text{ else } b]$$

The idea is to infer the dynamic of the intervals thanks to the first terms.

## Computing inductive invariants as intervals - 3

The widening operator being designed, we compute  $(x \subseteq F(x))$

$$\Sigma_0, Y_1 = \Sigma_0 \nabla F(\Sigma_0), Y_2 = Y_1 \nabla F(Y_1) \dots$$

**finite computation** instead of :

$$\Sigma_0, F(\Sigma_0), F^2(\Sigma_0), \dots$$

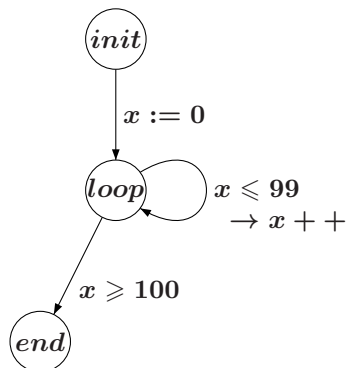
which can be infinite.

### Theorem

*(Cousot/Cousot 77) Iteratively computing the reachable states from the entry point with the interval operators and applying widening at entry nodes of loops converges in a **finite** number of steps to a overapproximation of the least invariant (aka **postfixpoint**).*

- ▶ The widening operators must satisfy the non ascending chain condition (see Cousot/Cousot 1977).

# Invariants for programs - ex 1



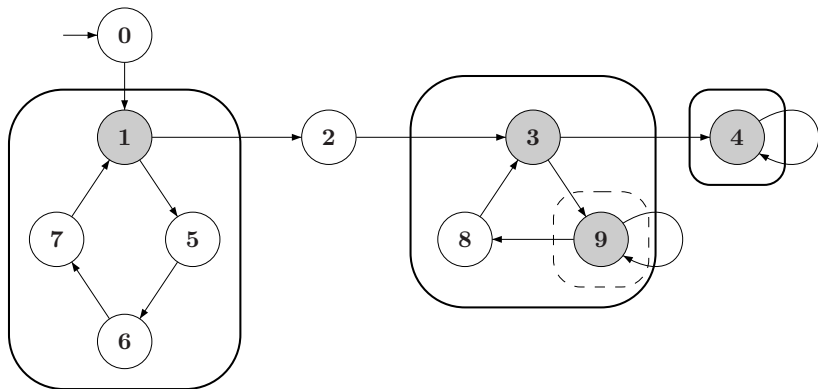
- ▶  $x \in [0, +\infty]$  in loop.

## Computing inductive invariants as intervals - ex 2

```
x = random(0,7);  
y = cos(x)+x  
while (y<=100) {  
  if (x>2) x--;  
  else {  
    y = -4;  
    x--;  
  }  
}
```

## Nested loops / Several loops

(Bourdoncle, 1992) Computing strongly connected subcomponents and iterate inside each :



Gray nodes are **widening nodes**

# Improving precision after convergence - 1

```
int x=0; /* [0, 0] */
while /* [0, +infty) */ (x<1000) {
  /* [0, 999] */
  x=x+1;
  /* [1, 1000] */
}
```

we got  $[0, +\infty)$  instead of  $[0, 999]$ . Run one more iteration of the loop :  $\{0\} \sqcup [1, 1000] = [0, 1000]$ . Check if  $[0, 1000]$  is an inductive invariant? **YES**

► This is called **narrowing** or descending sequence : ends when we have an inductive invariant or after  $k$  applications of the transition function.

## Improving precision after convergence - 2

Let  $\hat{x}$  be the result of the computation

### Result

The descending sequence always improves precision.

**Proof :**  $lfp(F) \subseteq \hat{x}$ , then  $F(lfp(F)) = lfp(F) \subseteq F(\hat{x})$ , and  $F(\hat{x})$  is again a correct invariant. If  $\hat{x}$  is not a fixpoint, then  $F(\hat{x}) \subset \hat{x}$ , so is a strictly better invariant.

# Best invariant in domain not computable

$P()$  ;

$x=0$  ;

Best invariant at end of program, as interval ?

$[0, 0]$  iff  $P()$  terminates

$\emptyset$  iff  $P()$  does not terminate

Entails solving the **halting problem**.

## When intervals are not sufficient

```
assume(x >= 0 && x <= 1);  
y = x;  
z = x-y;
```

- The human (intelligent) sees  $z = 0$  thus interval  $[0, 0]$ , taking into account  $y = x$ .
- Interval arithmetic does not see  $z = 0$  because it does not take  $y = x$  into account.

# How to track relations

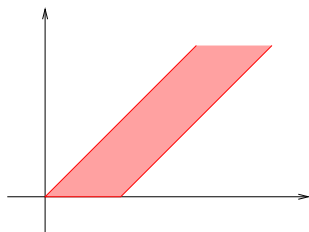
Using **relational domains**.

E.g. : keep

- for each variable an interval
- for each pair of variables  $(x, y)$  an information  $x - y \leq C$ .
- (One obtains  $x = y$  by  $x - y \leq 0$  and  $y - x \leq 0$ .)

How to **compute** on that ?

## Bounds on differences : practical example



Suppose  $x - y \leq 4$ , computation is  $z = x + 3$ , then we know  $z - y \leq 7$ .

Suppose  $x - z \leq 20$ , that  $x - y \leq 4$  and that  $y - z \leq 6$ , then we know  $x - z \leq 10$ .

We know how to **compute** on these relations (transitive closure / shortest path).

On our example, obtain  $z = 0$ .

## Why this is useful

Let  $t(0..n)$  an array in the program.

The program writes  $t(i)$ .

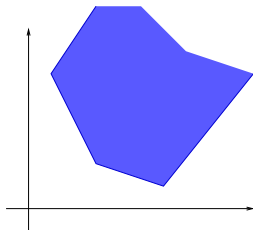
Need to know whether  $0 \leq i \leq n$ , otherwise said find bounds on  $i$  and on  $n - i \dots$

## Can we do better ?

How about tracking relations such as  $2x + 3y \leq 6$  ?

At a given program point, a set of **linear inequalities**.

In other words, a **convex polyhedron**.



- 1 Data Flow analysis
- 2 Abstract Interpretation
- 3 **Linear Relation Analysis**
  - Classical Linear Relation Analysis
  - Some improvements
  - Diverse use of AI
- 4 And ?

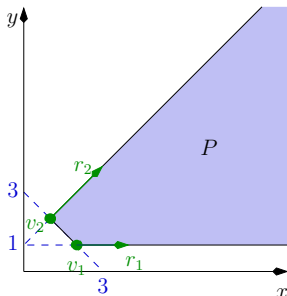
# Intro

(Halbwachs/Cousot 1979)

- Abstract Interpretation in the Polyhedral domain
- Infinite Domain with many particularities
- Discover affine relations on variables
- ▶ Classically used in verification problems.

# The polyhedral domain (1)

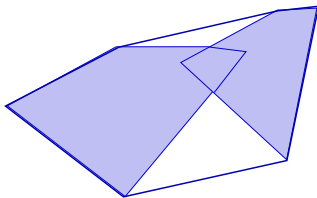
Convex polyhedra representation :



- ▶ Effective and efficient algorithmic (emptiness test, union, affine transformation . . .)

## The polyhedral domain(2)

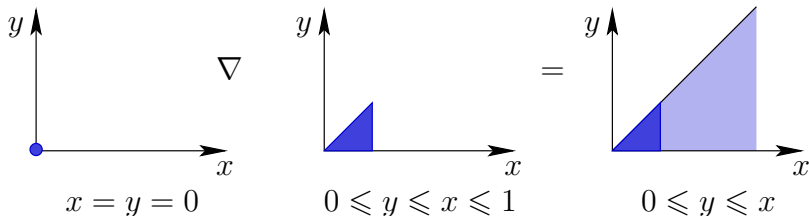
- Intersection, emptiness
- Affine Transformation :  $a(P) = \{CX + D \mid X \in P\}$ .
- Convex union (loss of precision)



## The Polyhedral domain (3)

**Widening** :  $P \nabla Q$  : limit extrapolation.

$P \nabla Q$  constraints : take  $Q$  constraints and remove those which are not saturated by  $P$ .



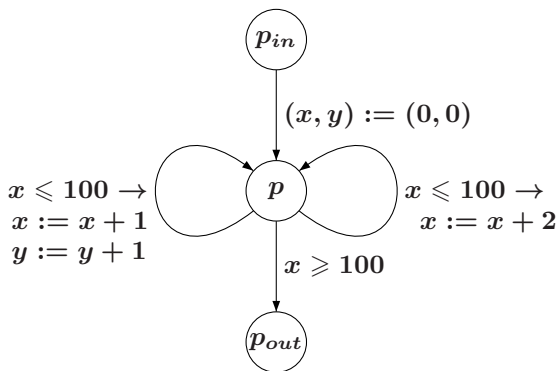
Trick (!) :  $\{x = y = 0\} = \{0 \leq y \leq x \leq 0\}$

## Analysis example - 1

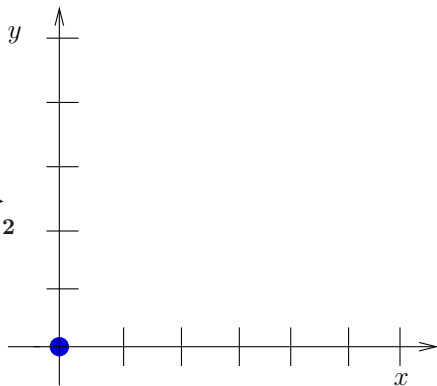
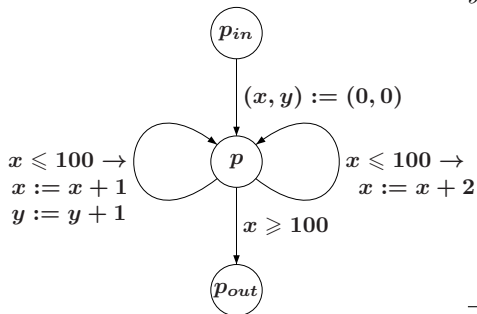
```

x:=0;y:=0
while (x<=100) do
  read(b);
  if b then
    x:=x+2
  else begin
    x:=x+1;
    y:=y+1;
  end;
endif
endwhile

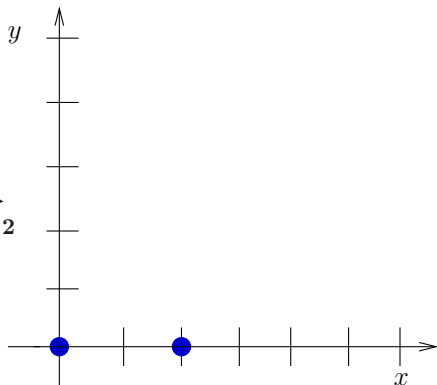
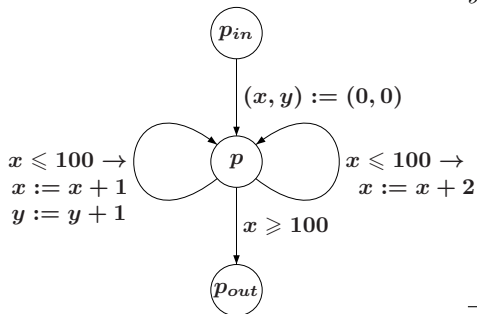
```



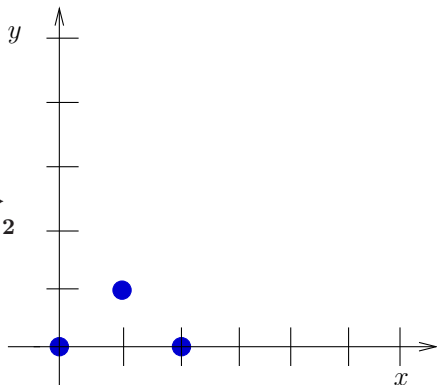
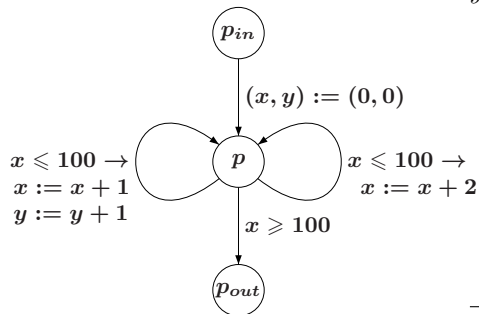
# Example - 2



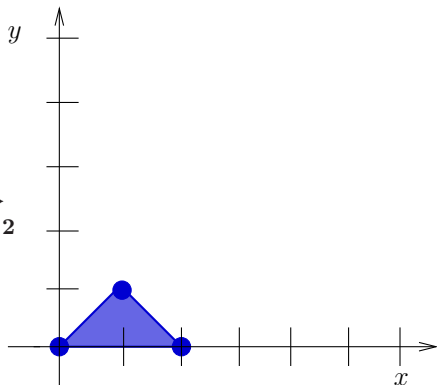
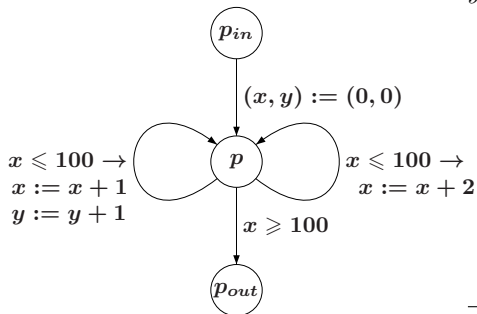
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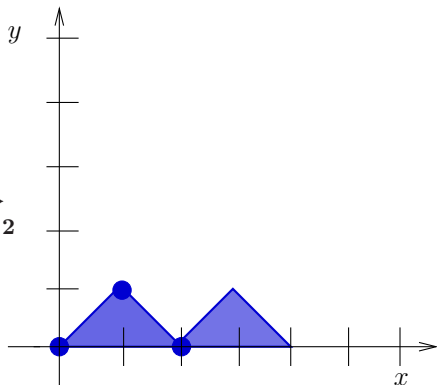
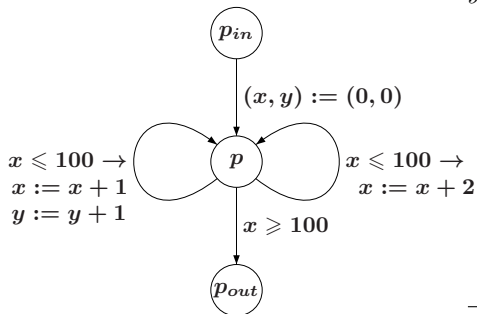
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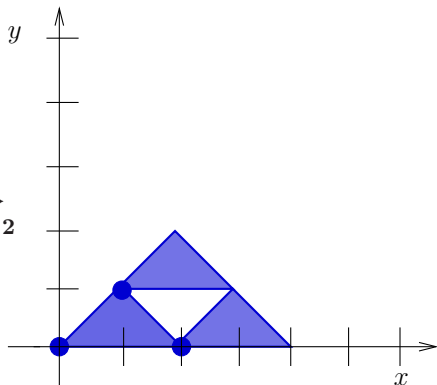
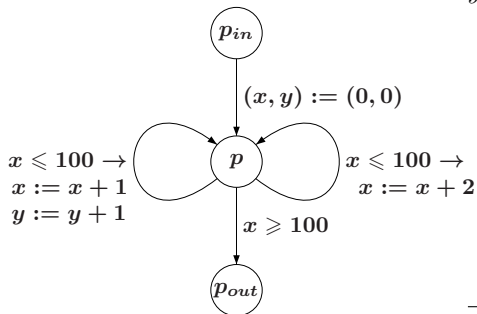
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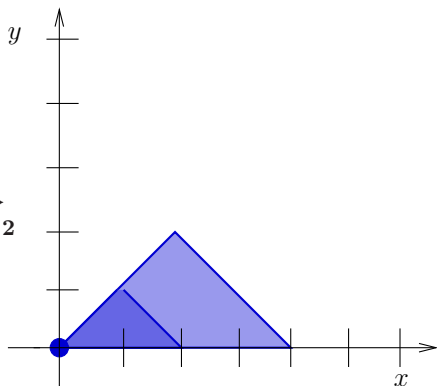
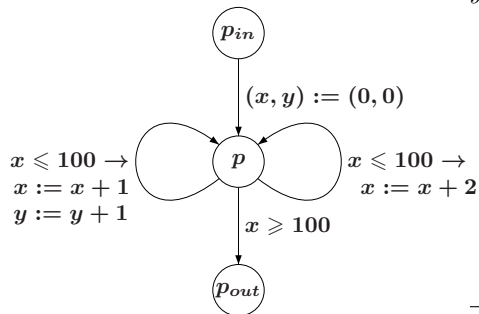
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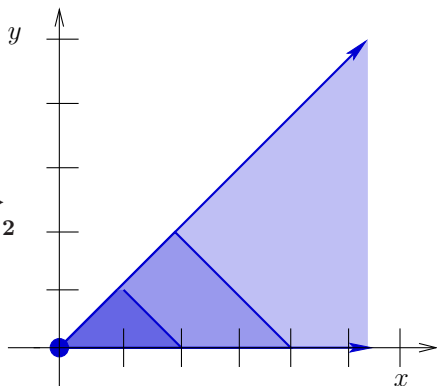
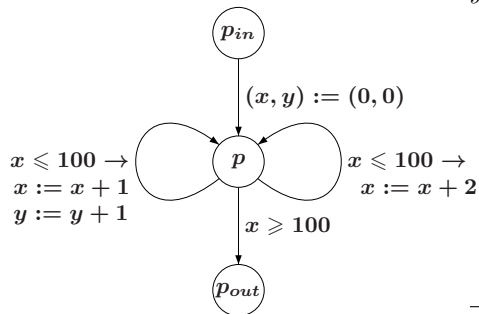
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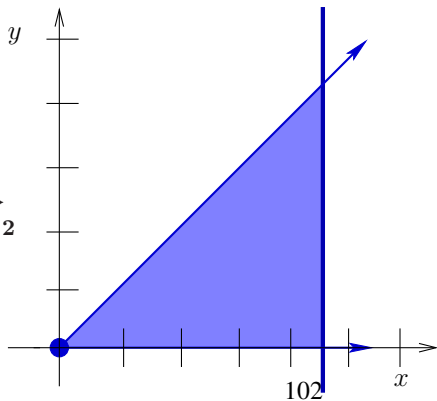
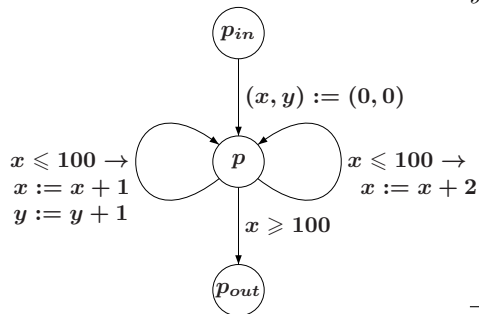
# Example - 2



# Example - 2



# Example - 2



# Linear Relation Analysis - Problems

Complexity increases with :

- number of control points
- number of numerical variables

Approximation is due to :

- Convex hulls
- Widening

(credits for these slides : Nicolas Halbwachs)

# Complexity

(In general) The more precise we are, the higher the costs.  
For each line of code :

- Intervals : algorithms  $O(n)$ ,  $n$  number of variables.
- Differences  $x - y \leq C$  : algorithms  $O(n^3)$
- Octagons  $\pm x \pm y \leq C$  (Miné) : algorithms  $O(n^3)$
- Polyhedra (Cousot / Halbwachs) : algorithms often  $O(2^n)$ .

# Delaying widening - 1

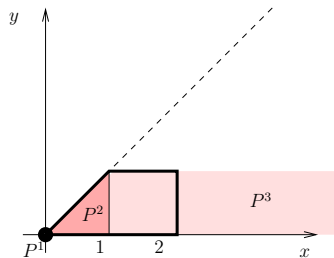
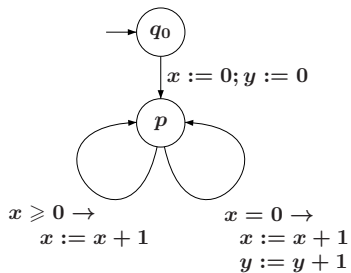
Halbwachs 1993/Goubault 2001 / Blanchet 2003

Fix  $k$  and compute :

$$X_n = \begin{cases} \perp & \text{if } n = 0 \\ F(X_{n-1}) & \text{if } n < k \\ X_{n-1} \nabla F(X_{n-1}) & \text{else.} \end{cases}$$

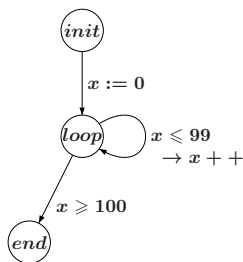
- ▶ Similar to unrolling loops, costly but useful (regular behaviour after a constant number of iterations).

# Delaying widening - 2 - ex



## Improving the widening operator

While applying  $P \nabla Q$ , intersect with constraints that are satisfied by both  $P$  and  $Q$ . The constraints must be precomputed.



Here, with “ $x \leq 100$ ” in the pool of constraints, it avoids narrowing.

► Warning **widening is not monotone**, so improving locally is not necessarily a good idea !

# Local improvement with acceleration

(Gonnord/Halbwachs 2006)

**Idea** : Sometimes, a fixpoint of a loop can be easily computed without any fixpoint iteration.

[More details here](#)

# Good path heuristic

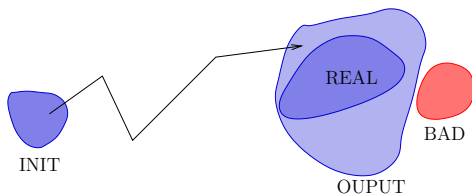
(Gonnord/Monniaux 2011)

**Idea** : find interesting paths by means of smt-queries

[More details here](#)

# Applications

- Bounds on iterators of arrays (intervals, differences on bounds)
- Dead code elimination (all domains) - especially when the code has been automatically generated / asserts
- Vectorization : computations that can be permuted
- Memory optimisation : this `int` can be encoded in 16 bits ?
- Safety analysis



- 1 Data Flow analysis
- 2 Abstract Interpretation
- 3 Linear Relation Analysis
- 4 And ?

# Tools - 1

ASPIC : **A**ccelerated **S**ymbolic **P**olyhedral **I**nvariant  
**C**omputation

Aspic is **an invariant generator** :

- From counter automata with numerical variables.
- Invariants are **polyhedra**.

C2fsm is **a C parser** :

- From a source file in (a subset of) C into Aspic input language (fast).
- **Safe** abstractions of non numerical variables, structures, behaviors.

▶ <http://laure.gonnord.org/pro/aspic/aspic.html>

## Tools - 2

- Frama-C : analysing/ proving correction of C programs  
(see <http://frama-c.com/>)
- Apron : numerical domain interface  
(<http://apron.cri.enscm.fr/library/>)
- Interproc : IA analyser connected to Apron (see <http://pop-art.inrialpes.fr/interproc/interprocweb.cgi>)
- Rose / LLVM : C (and more) parsers and API for manipulating C programs. Rose is more decicated to program transformation, LLVM to compiler construction(<http://www.rosecompiler.org/> and <http://llvm.org/>).

# Industrial succes stories

- Polyspace
- Astree

## Other analyses

- WCET computation by means of LP
- Cache analysis with AI (Maiza, Reineke, Wilhelm . . .)

The End.

Thanks !

# LRA and acceleration

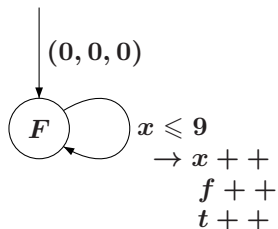
(SAS 2006, Phd 2007)

## **Combination** LRA and acceleration techniques [Finkel/Sutre/Leroux/...]

- Abstract acceleration notion :
  - low-cost overapproximations ;
  - inside LRA, combination with widening.
- Classification of accelerable loops.
- Prototype : ASPIC

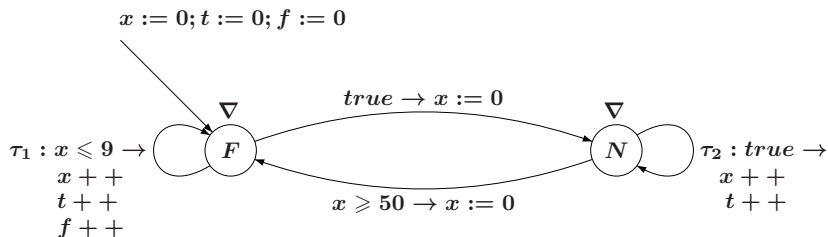
# Acceleration - accelerable loops

## An easy case



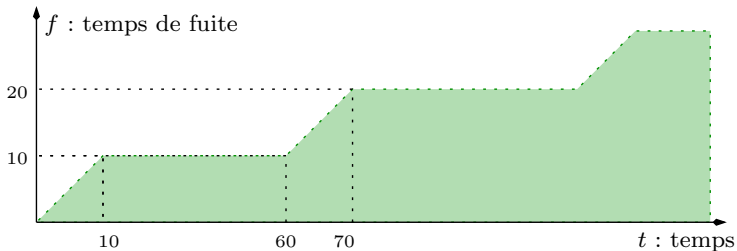
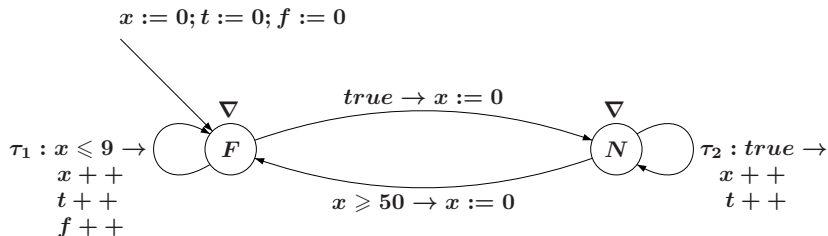
- ▶ **exact effect** :  $\exists i \in \mathbb{N}, x = f = t = i, 0 \leq i \leq 10$
- ▶ **exact effect in the abstract domain** :  
 $\{x = f = t, 0 \leq t \leq 10\}$

# Gas Burner example - 1

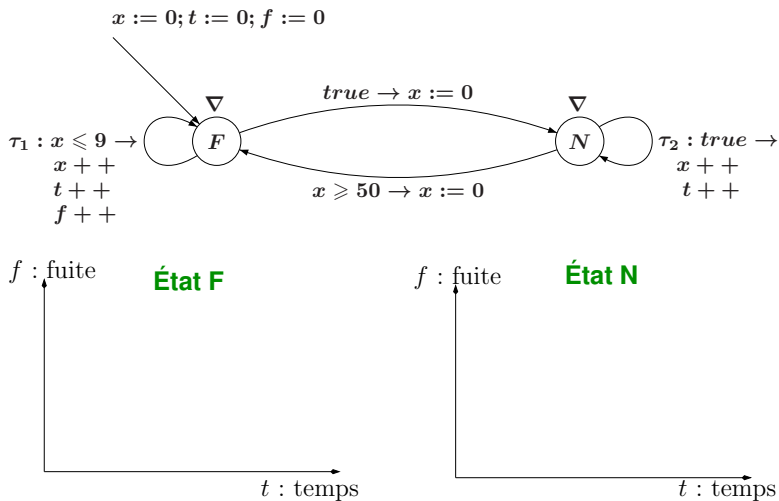


- $f$  global leaking time
- $t$  global time
- $x$  local variable

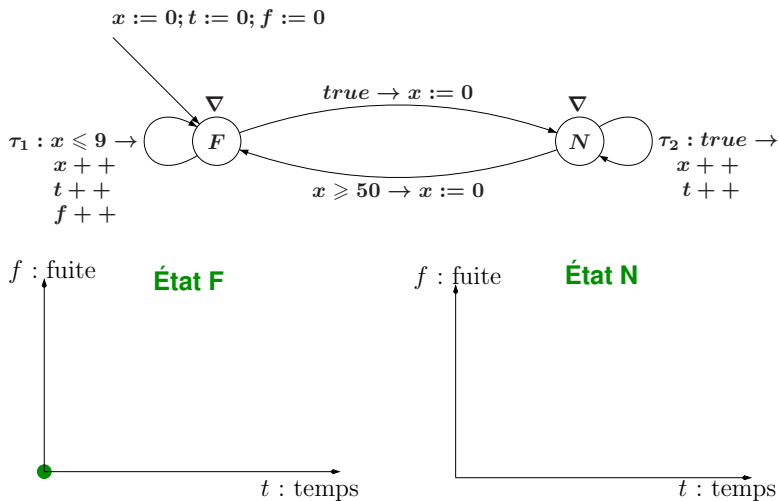
# Gas burner 2 - Real Behaviour



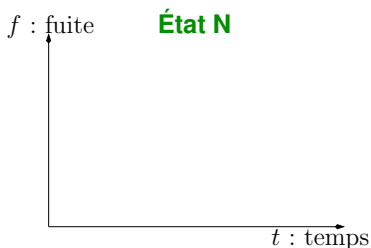
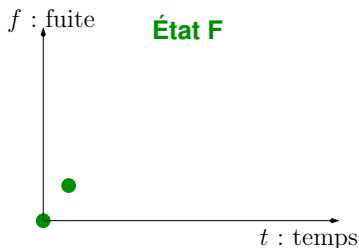
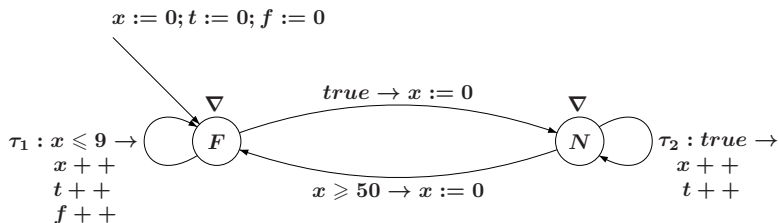
## Gas burner 3 - with LRA



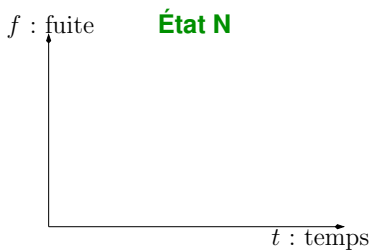
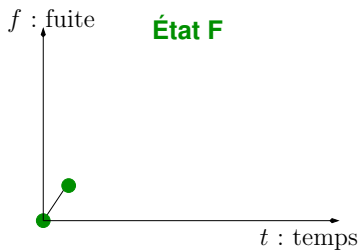
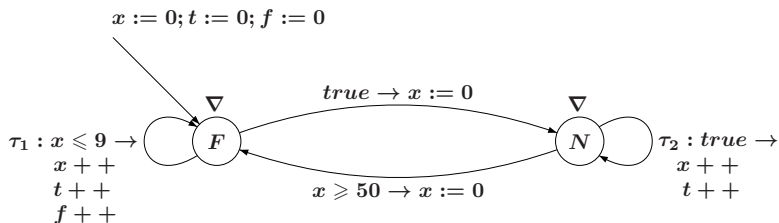
## Gas burner 3 - with LRA



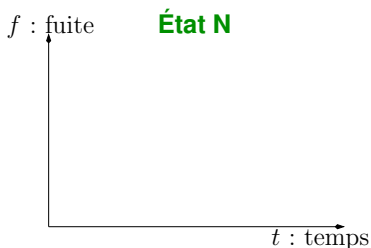
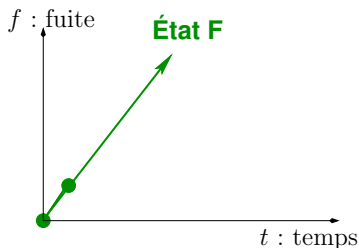
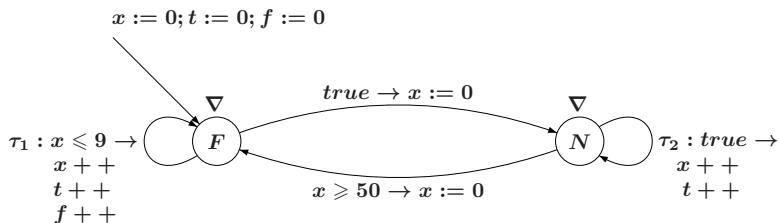
## Gas burner 3 - with LRA



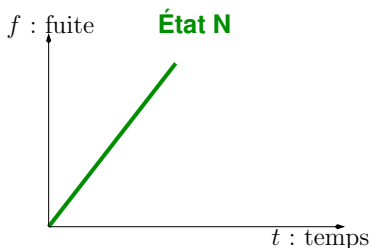
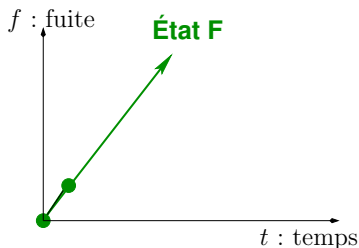
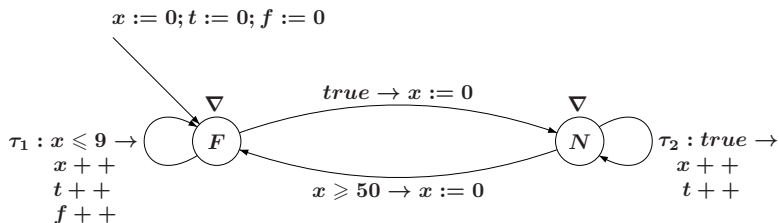
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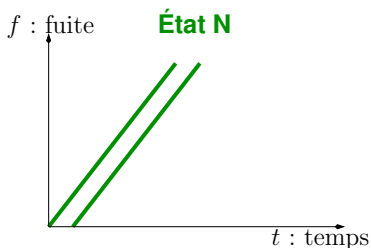
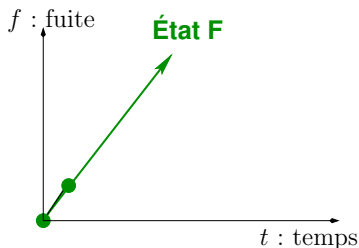
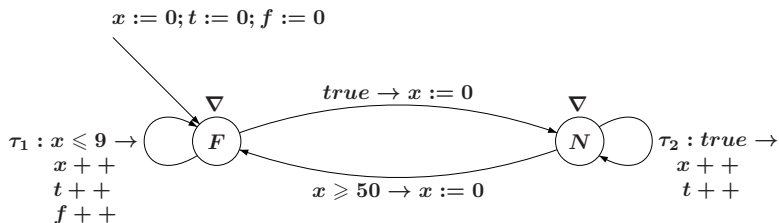
## Gas burner 3 - with LRA



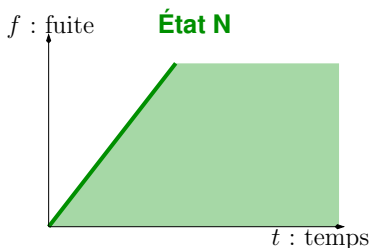
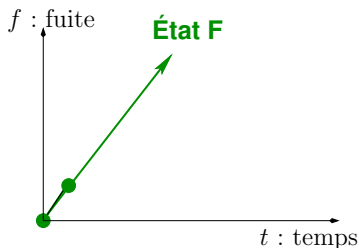
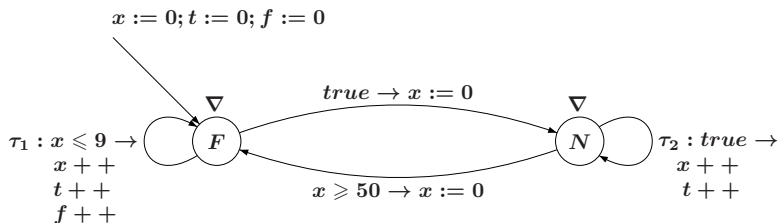
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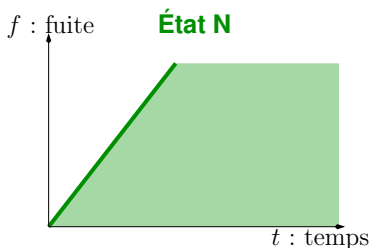
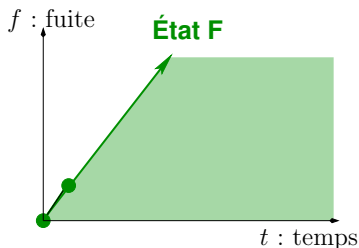
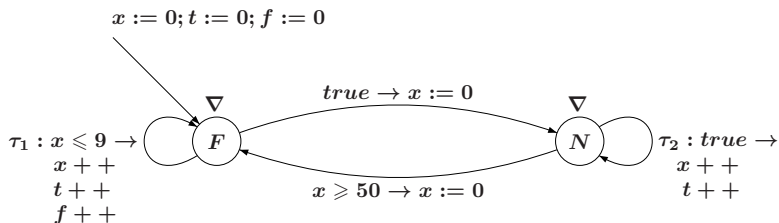
## Gas burner 3 - with LRA



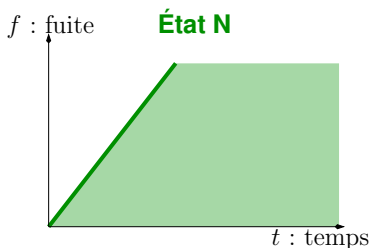
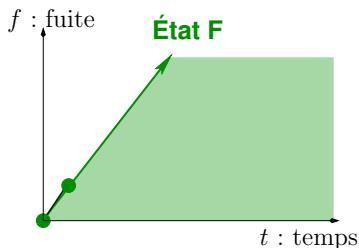
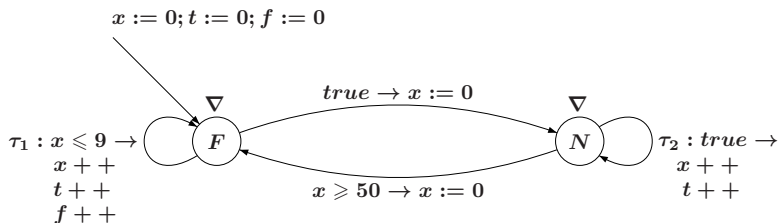
## Gas burner 3 - with LRA



## Gas burner 3 - with LRA

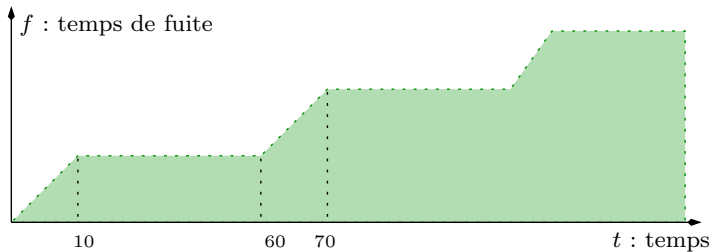
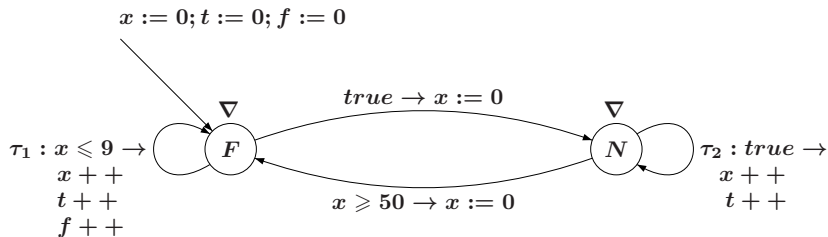


## Gas burner 3 - with LRA

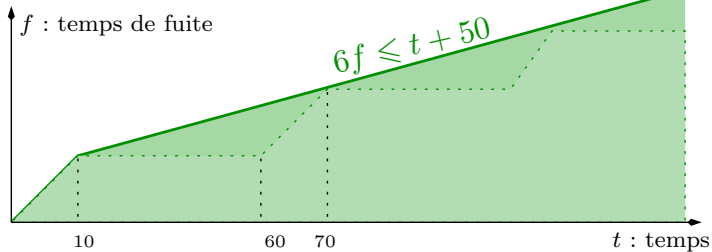
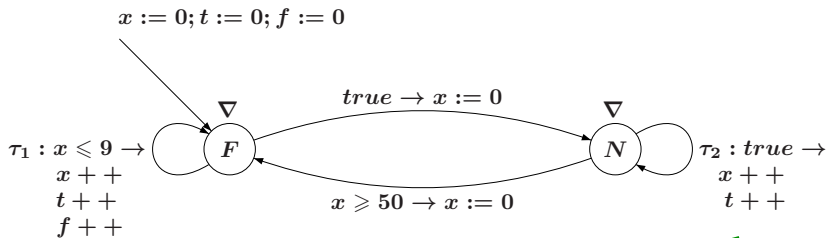


► Loss of precision

# Gas burner - desired invariant

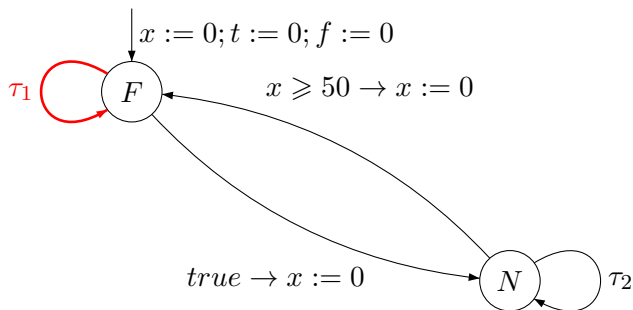


# Gas burner - desired invariant



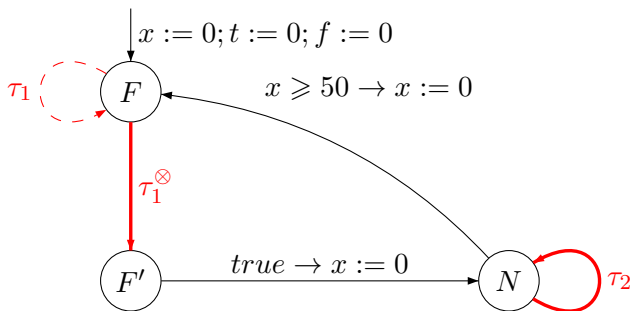
# Accelerating the gas burner - 1

(mini-)loops are replaced ( $\tau_i : g_i \rightarrow a_i$ )



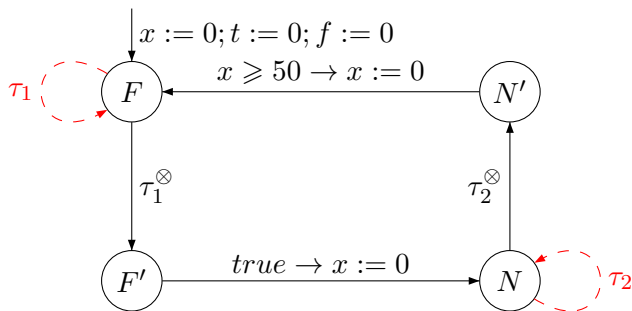
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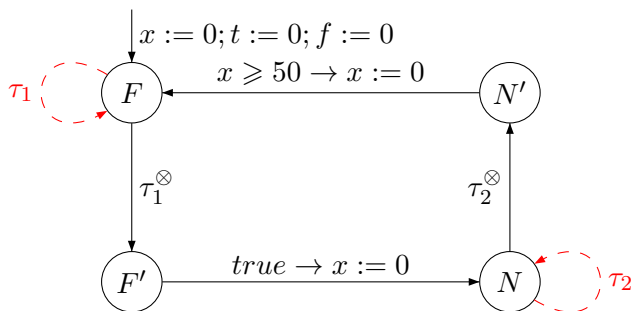
# Accelerating the gas burner - 1

(mini-)loops are replaced ( $\tau_i : g_i \rightarrow a_i$ )



# Accelerating the gas burner - 1

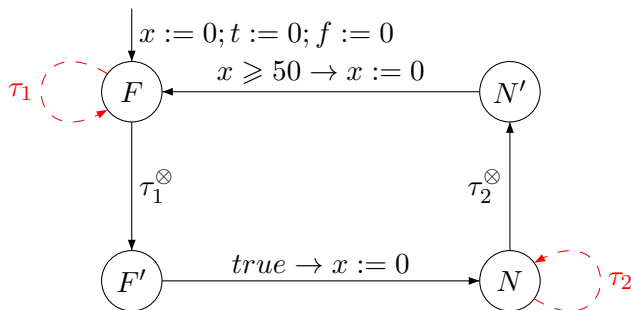
(mini-)loops are replaced ( $\tau_i : g_i \rightarrow a_i$ )



►  $\tau_i^\otimes$  summarizes the effect of any application of  $\tau_i$  (unfixed number of iterations).

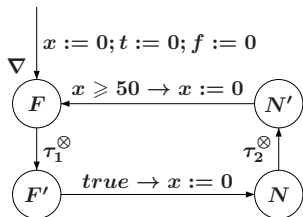
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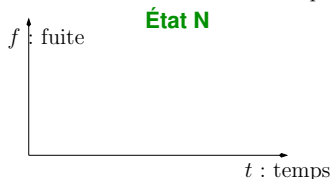
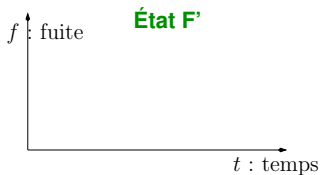
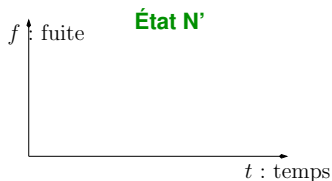
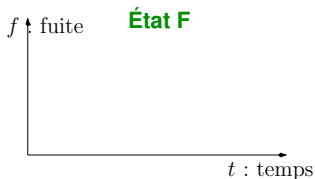


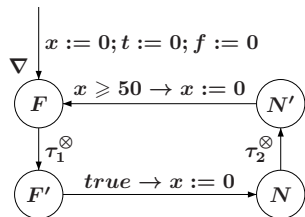
►  $\tau_i^\otimes$  summarizes the effect of any application of  $\tau_i$  (unfixed number of iterations).

► Outer loop **widened**.

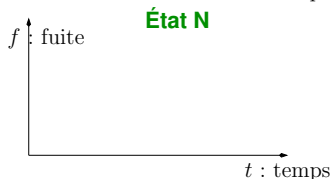
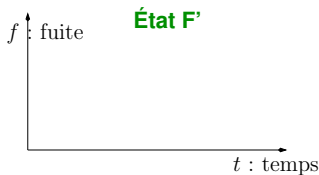
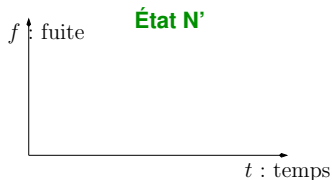
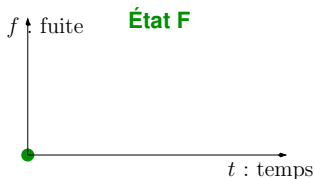
Accelerated Gas Burner - 2 back

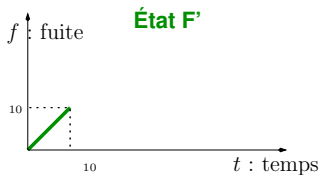
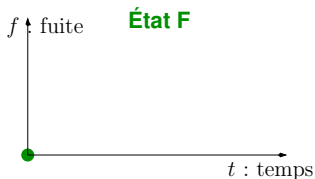
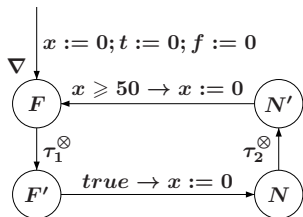
- $\tau_1^\otimes$  = “add ray (1, 1, 1)  
while  $x \leq 10$ ”
- $\tau_2^\otimes$  = “add ray (1, 0, 1)”



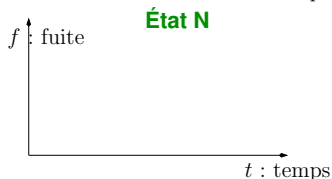
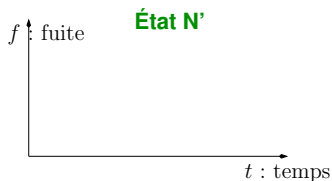
Accelerated Gas Burner - 2 back

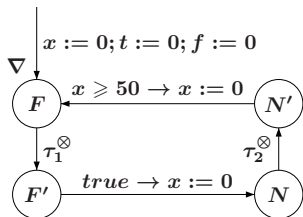
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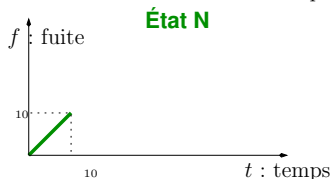
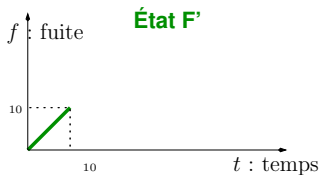
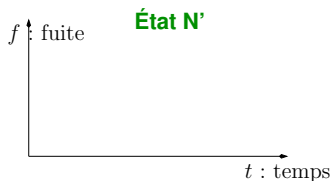
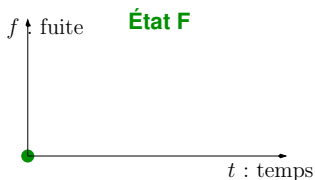
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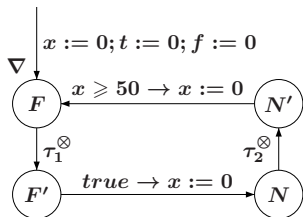
- $\tau_1^\otimes = \text{"add ray (1, 1, 1) while } x \leq 10\text{"}$
- $\tau_2^\otimes = \text{"add ray (1, 0, 1)"}$



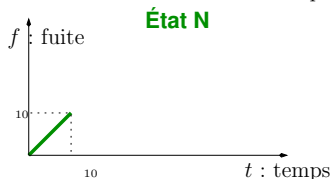
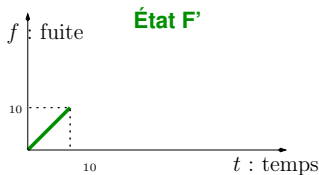
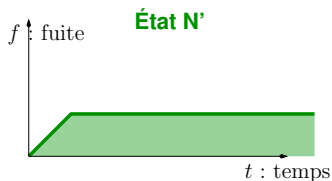
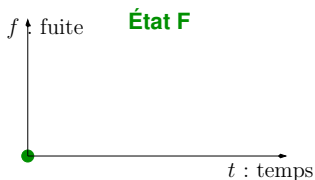
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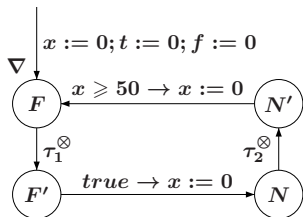
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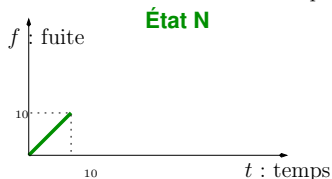
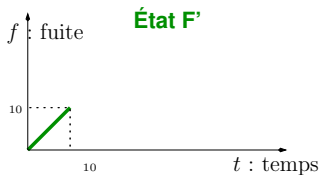
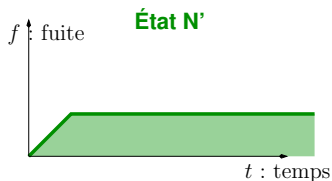
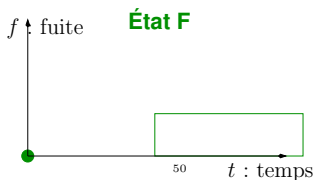
Accelerated Gas Burner - 2 back

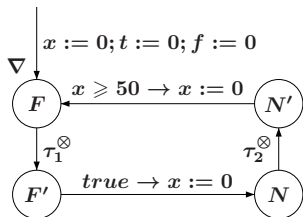
- $\tau_1^\otimes = \text{"add ray (1, 1, 1) while } x \leq 10\text{"}$
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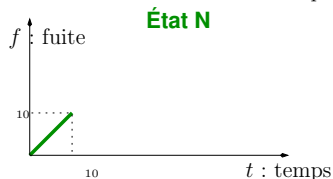
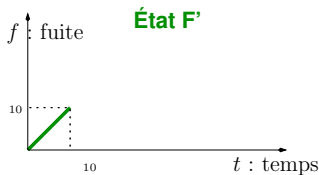
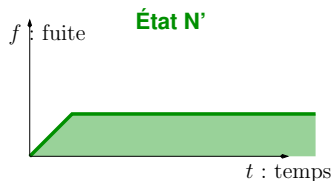
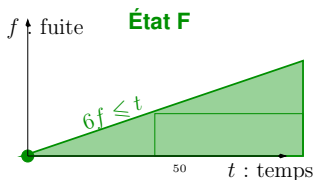
Accelerated Gas Burner - 2 back

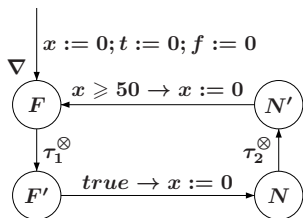
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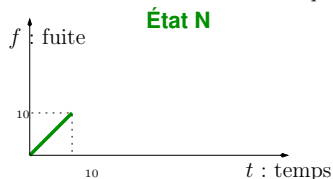
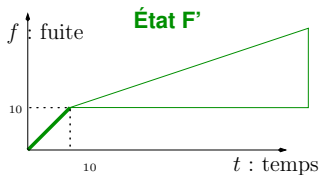
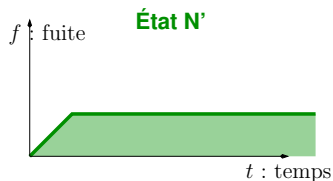
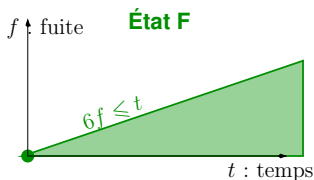
Accelerated Gas Burner - 2 back

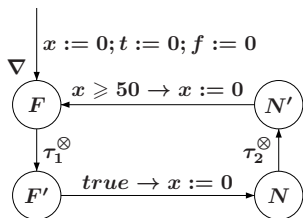
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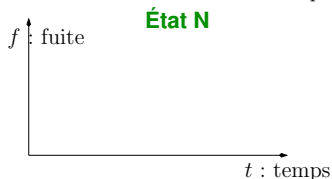
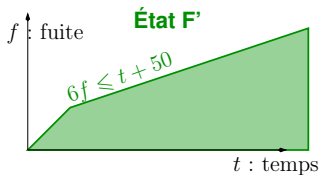
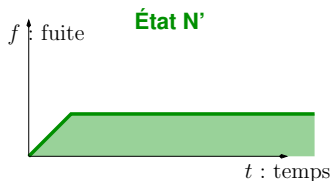
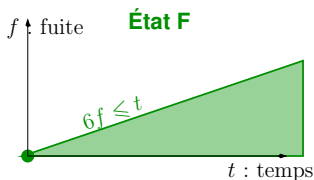
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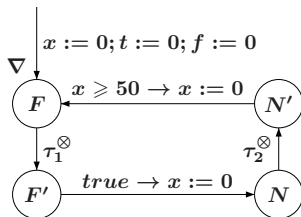
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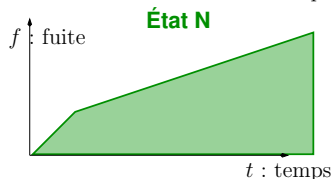
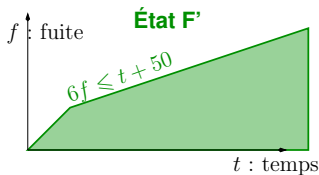
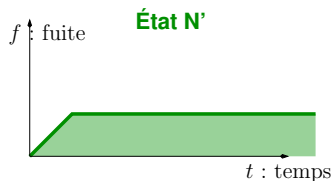
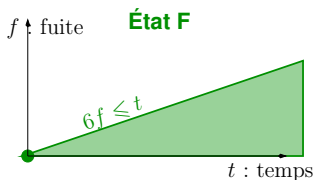
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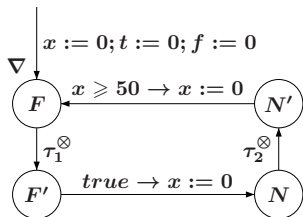
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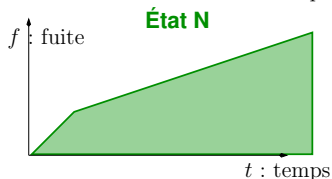
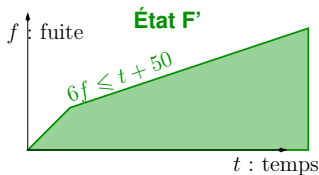
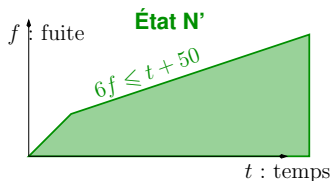
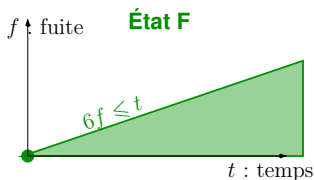
Accelerated Gas Burner - 2 back

- $\tau_1^{\otimes}$  = “add ray (1, 1, 1)  
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Accelerated Gas Burner - 2 back

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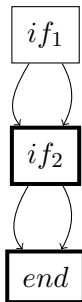


# SMT+LRA, Motivation : example 1

Some properties cannot be expressed in convex abstract domains :

```

if (x >= 0) { xabs = x; }
  else { xabs = -x; }
if (xabs >= 0.01) {
  y = (sin(x) / x) - 1;
} else {
  xsq = x*x;
y = xsq*(-1/6. + xsq/120.);
}
  
```



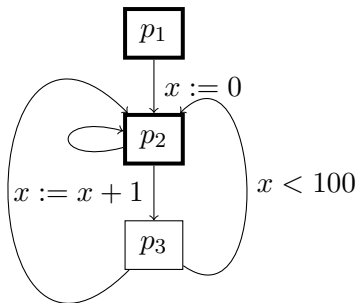
- Store the fact that  $x = xabs \vee x = -xabs$  at node *if2*

## SMT+ LRA, Motivation : example 2

The widening operator can be too coarse :

```
int x = 0;
while (true) {
  if (nondet()) {
    x = x+1;
    if (x >= 100) x = 0;
  }
}
```

$x \geq 100$   
 $x := 0$

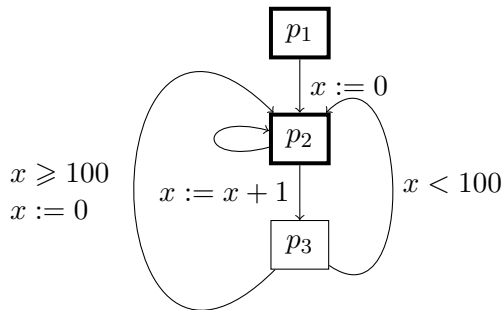


- The analysis (interval domain) gives  $[0, +\infty)$ , not improved by narrowing !

## Two ideas

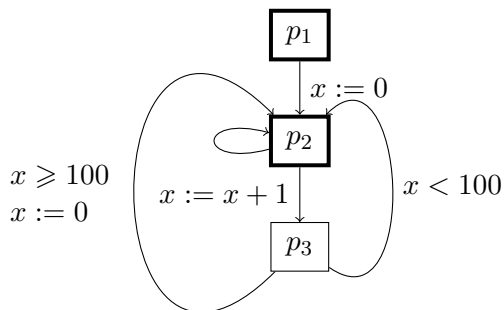
- First idea : do not compute the convex hull on “diamonds”.
  - Second idea : consider all paths and analyse them **separately**.
  
  - Advantage : precision
  - Drawback : combinatorial explosion
- ▶ Our method will implement these two ideas without computing all the program paths explicitly.

# Invariants



Is  $x = 0$  an invariant in  $p_2$  ?

# Invariants

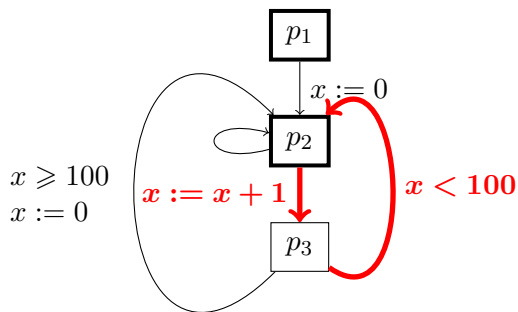


Is  $x = 0$  an invariant in  $p_2$  ?

► No ! Because it's not stable.

## Our method on example 2

Focus on the red path !



- ▶ Its least inductive invariant (for  $p_2$ ) is  $x \in [0, 99]$ , which is also an invariant while considering the whole graph.

## How to detect paths ? [back](#)

We delegate the search for new paths to an SMT solver.  
The problem is encoded into an SMT-problem thanks to the use of an internal **structure** :

- compact ; (complexity reasons)
  - acyclic ; (to reason about loops as for paths)
  - all variables are assigned once (to reason about unique variable values).
- ▶ Preprocessing : computing this structure.  
(SAS 2011 for technical details)